May 4th, 2018

Math 241

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Spring 2018

Name: ....

Student ID (last 6 digits): XXX-.................

Final

Version A

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 8 questions and 18 printed pages (including this one and *two* pages for scratch work in the end).

No calculators, books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	11	
2	16	
3	10	
4	12	
5	15	
6	9	
7	5	
8	21	
Total	100	

1) Let $\mathcal{C}$ be the curve given by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ for $t \in [0, 2]$.

(a) [3 points] Find the equation of the line tangent to $\mathcal{C}$ at the point given by $t = 1$.

(b) [3 points] Give a simple [i.e., Calculus 2] integral that gives the arc length of $\mathcal{C}$ for $t \in [0, 2]$. **Do not compute the integral!**

Continues on the next page!

(c) [5 points] Give a simple [i.e., Calculus 2] integral that gives the work against $\mathbf{F} = \langle x^2, -y^2, z \rangle$ of moving a particle along the curve $\mathcal{C}$ [i.e., the curve given by $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$ for $t \in [0, 2]$; to be clear, we are moving the particle starting at $t = 0$ and ending at $t = 2$]. **Do not compute the integral!**

2) Let $f(x, y) = xy$ and $\mathcal{D}$ be the region given by $4x^2 + 9y^2 \leq 32$.

(a) [2 points] Compute the partial derivatives f_x and f_y.

(b) [2 points] Find all the critical points of $f(x, y) = xy$ in $\mathbb{R}^2$. [So, not only inside $\mathcal{D}$.]

(c) [3 points] For each critical point of the previous part, classify it as local maximum, local minimum, or saddle.

Continues on the next page!

(d) [6 points] Use Lagrange's Multipliers to find the global maximum and minimum of $f(x, y) = xy$ on the ellipse $4x^2 + 9y^2 = 32$ [the boundary of $\mathcal{D}$].

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(e) [3 points] Find the global maximum and minimum of $f(x, y) = xy$ in $\mathcal{D}$ [the region given by $4x^2 + 9y^2 \leq 32$].

3) Let $f(x, y, z) = xy^2 - zx^2$ and $\mathbf{F} = \langle y^2 - 2xz, 2xy, -x^2 \rangle$.

(a) [3 points] Show that $f(x, y, z)$ is the potential of $\mathbf{F}$.

(b) [3 points] In what direction from $P = (0, 1, 0)$ does the potential of $\mathbf{F} = \langle y^2 - 2xz, 2xy, -x^2 \rangle$ [as before] increase the most? [Your answer should be a three dimensional vector. **Hint:** Don't let all the terminology confuse you. This is a very simple question.]

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(c) [4 points] Let $\mathcal{C}$ be the polygonal path [i.e., made of straight line segments] going from $(0, 0, 0)$, to $(0, -1, 0)$, to $(0, 0, 2)$ and finally to $(1, 1, 0)$. Compute $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$.

4) Let $\mathcal{C}$ be the triangle with vertices $(0,1)$, $(0,-1)$ and $(1,0)$, oriented *clockwise*, and $\mathbf{F} = \langle \mathrm{e}^x, \sin(y) - 2x \rangle$.

(a) [3 points] Sketch $\mathcal{C}$. Draw arrows on $\mathcal{C}$ to show the correct orientation.

(b) [3 points] Compute $\mathrm{curl}_z(\mathbf{F})$. [Remember $\mathbf{F} = \langle \mathrm{e}^x, \sin(y) - 2x \rangle$.]

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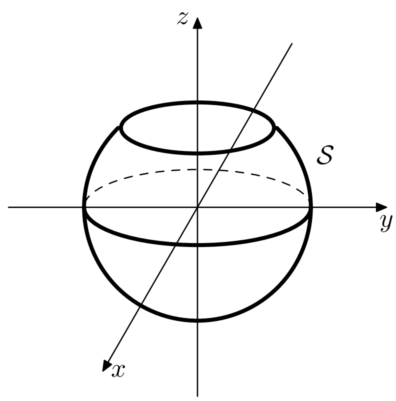
(c) [6 points] Compute $\oint_{\mathcal{C}} \mathbf{F} \cdot \mathrm{d}\mathbf{r}$. [Remember $\mathcal{C}$ is the triangle with vertices $(0,1)$, $(0,-1)$ and $(1,0)$, oriented *clockwise*, and $\mathbf{F} = \langle \mathrm{e}^x, \sin(y) - 2x \rangle$.]

[**Hint:** There is an easy way and a hard way of doing this. If you do it the hard way, you might be pressed on time.]

5) Let $\mathcal{S}$ be the surface given part of the sphere $x^2 + y^2 + z^2 = 13$ with $z \leq 2$ [so we are "chopping" the top of the sphere at height 2 – see figure below], oriented with normal vectors pointing *toward* its center. Let also $\mathbf{F} = \langle y, -x, 0 \rangle$



(a) [2 points] Draw on the picture above an arrow on the curve $\partial\mathcal{S}$ [the *boundary* of the surface $\mathcal{S}$ above] to show the correct boundary orientation.

[**Hint:** The boundary is where $z = 2$.]

(b) [4 points] Give a parametrization $\mathbf{r}(t)$ of $\partial\mathcal{S}$. [Don't forget to give the range of the parameter t.]

(c) [3 points] Compute curl($\mathbf{F}$). [Remember: $\mathbf{F} = \langle y, -x, 0 \rangle$.]

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(d) [6 points] Compute $\iint_{\mathcal{S}} \operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$. [Remember: $\mathbf{F} = \langle y, -x, 0 \rangle$.]

[**Hint:** There is an easy way and a hard way of doing this. If you do it the hard way, you might be pressed on time. Also, if you need to use (b), and could not find it, use the parametrization $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), 1 \rangle$ for $t \in [0, \pi]$.]

6) Let $\mathcal{W}$ be the box given by $0 \le x \le 1$, $0 \le y \le 2$, and $1 \le z \le 3$ [i.e., $[0,1] \times [0,2] \times [1,3]$] and $\mathbf{F} = \langle xy, yz, y^2 z \rangle$.

(a) [3 points] Compute div($\mathbf{F}$).

(b) [6 points] Compute the flux of $\mathbf{F}$ through [or across] $\partial \mathcal{W}$ with the usual boundary orientation.

[**Hint:** There is an easy way and a hard way of doing this. If you do it the hard way, you might be pressed on time.]

7) [5 points] Let $\mathcal{S}$ be the surface given by the parts of the sphere $x^2 + y^2 + z^2 = 9$ with $x \geq 0$ and $z \leq 0$. Give a parametrization of $\mathcal{S}$.

8) Let $\mathcal{S}$ be the surface given by the parametrization $G(u, v) = (u, v, u^2 - v^2)$ for $(u, v) \in \mathcal{D}$, where $\mathcal{D}$ is the disc $x^2 + y^2 \leq 4$.

(a) [3 points] Compute the partial derivatives $G_u(u, v)$ and $G_v(u, v)$.

(b) [3 points] Compute $G_u(u, v) \times G_v(u, v)$ [for G_u and G_v found in part (b)].

(c) [3 points] Give the equation of the plane tangent to $\mathcal{S}$ at the point $(0, 1, -1)$ in the form $ax + by + cz = d$.

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(d) [6 points] Express the surface area of $\mathcal{S}$ [the same $\mathcal{S}$ as in the previous items] as iterated simple [i.e., Calculus 2] integrals. [In other words, you will set up the integral that gives the area of $\mathcal{S}$, **but not compute it!**]

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(e) [6 points] Let now $\mathbf{F} = \langle z, y, xy \rangle$ and $\mathcal{S}$ be as above [given by $G(u,v) = (u, v, u^2 - v^2)$ for $(u,v) \in \mathcal{D}$, where $\mathcal{D}$ is the disc $x^2 + y^2 \leq 4$] oriented with *upward pointing* normal vectors. Express $\iint_{\mathcal{S}} \mathbf{F} \cdot \mathrm{d}\mathbf{S}$ as a double integral. [In other words, **you do *not* even need to write it as iterated simple integrals**, simply as a *double* (Calculus 3, Chapter 15) area integral, something like $\iint_{\cdots} \cdots \, \mathrm{d}A$. *Don't forget to give the domain of integration and simplify the integrand!*]

Scratch:

Scratch: