March 27th, 2018

Math 241

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Spring 2018

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 3

Version A

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 9 printed pages (including this one and a page for scratch work in the end).

No calculators, books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	15	
2	20	
3	25	
4	20	
5	20	
Total	100	

1) [15 points] Let $f(x, y)$ be a function with continuous second order partial derivatives. The table below gives the list of all its critical points, as well as its second order partial derivatives at those points. Fill in the discriminant [also called the Hessian determinant] column and check the corresponding box if the second derivative test gives that the critical point is a local minimum, local maximum, saddle point or if the test is inconclusive.

P (critical point)	$f_{xx}(P)$	$f_{yy}(P)$	$f_{xy}(P)$	discriminant	classification
$(1, 2)$	2	1	-1		☐ loc. min. ☐ loc. max. ☐ saddle ☐ inconcl.
$(3, 1)$	-3	-1	2		☐ loc. min. ☐ loc. max. ☐ saddle ☐ inconcl.
$(0, -2)$	1	1	1		☐ loc. min. ☐ loc. max. ☐ saddle ☐ inconcl.
$(-3, 0)$	-4	-1	-1		☐ loc. min. ☐ loc. max. ☐ saddle ☐ inconcl.
$(-1, 2)$	0	2	-3		☐ loc. min. ☐ loc. max. ☐ saddle ☐ inconcl.

2) The temperature at a point (x, y) on the plane is give by the function $T(x, y) = 2x^2 + y^2$.

(a) [10 points] In what direction from the point $(1, 1)$ does the temperature *decrease* the most? [Your answer should be a vector giving the direction on the xy-plane.]

(b) [10 points] What is the rate of change of the temperature if you move from $(1, 1)$ again, but in the direction of the vector $\langle -1, 1 \rangle$?

3) Let $f(x, y) = 5 + 4xy - x^2 - y^2$.

(a) [8 points] Find all the critical points of $f(x, y)$. [*No need to classify them (as local maximum, minimum or saddle)!*]

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(b) [12 points] Use Lagrange multipliers to find the possible extrema of $f(x, y) = 5+4xy-x^2-y^2$ [same $f(x, y)$] with the constraint $x^2+y^2=4$. [*No need to classify them (as local maximum, minimum or saddle)!*]

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(c) [5 points]Find the *global* maximum and minimum of $f(x,y) = 5 + 4xy - x^2 - y^2$ [the same $f(x,y)$] on $x^2 + y^2 \leq 4$ and at which points they occur.

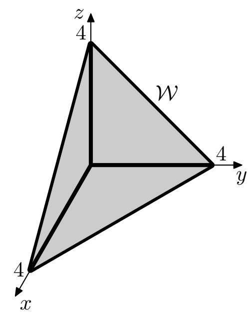
4) Consider the integral

$$\int_0^4 \left[\int_{y/2}^2 (x^2 - 1)^{2018}\,\mathrm{d}x \right] \mathrm{d}y.$$

(a) [10 points] Draw the region of integration for the double integral it represents.

(b) [10 points] Express the integral in the opposite order. **Do *not* compute the integral!**

5) [20 points] Let $\mathcal{W}$ be the solid tetrahedron [i.e., a pyramid with four faces] bounded by the planes $x = 0$ [the yz-plane], $y = 0$ [the xz-plane], $z = 0$ [the xy-plane] and $x + y + z = 4$. [See the image below]. Compute $\iiint_{\mathcal{W}} \mathrm{e}^z \, \mathrm{d}V$.



Scratch: