April 1st, 2015

Math 351

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Spring 2015

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 2 (Take Home)

You must turn in this exam in class on Monday, April 6th. Since this is a take home, I want all your solutions to be neat and well written.
Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 6 printed pages (including this one).

You can look at your notes, *our* book and solutions posted by me, but you cannot look at any other references (including the Internet) and you cannot discuss this with *anyone*!

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) Let R be a commutative ring [with $1 \neq 0$] and let

$$S = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \; : \; a, b \in R \right\}.$$

[So, diagonal 2×2 matrices with entries in R.]

(a) Prove that S is a ring [with the *usual* addition and multiplication of rings].

(b) Prove that R is *not* a domain.

2) Is $\mathbb{C}$ the field of fractions of $\mathbb{R}$? [Justify your answer!]

3) Prove that if R is a domain, then $U(R[x]) = U(R)$. [Remember, $U(R)$ is the set of units of R, which I usually denote by $R^\times$. So, what you need to prove it that the units of the polynomial ring are the constant polynomials which are units of R.]

4) Let F be a field having $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$ [or $\mathbb{F}_2 = \mathbb{I}_2$ with the notation from the book] as a subfield. Prove that for all $a \in F$ we have that $a + a = 0$ [i.e., $a = -a$].

5) Let F be a finite field with n elements and $a \in F \backslash \{0\}$. Prove that there is $k \in \{1, 2, \ldots, n\}$ such that $a^k = 1$. [**Hint:** Consider the set $S = \{1, a, a^2, a^3, \ldots, a^n\} \subseteq F$. How many *distinct* elements can S have?]