February 27th, 2015

Math 351

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Spring 2015

Name: ....

Student ID (last 6 digits): XXX-.................

Midterm 1 (Take Home)

You must turn in this exam in class on Friday, February 27th. Since this is a take home, I want all your solutions to be neat and well written.
Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 2 questions and 3 printed pages (including this one).

You can look at your notes, *our* book and solutions posted by me, but you cannot look at any other references (including the Internet) and you cannot discuss this with *anyone*!

Good luck!

Question	Max. Points	Score
1	14	
2	14	
Total	28	

1) [14 points] Prove that if $(a, b) = 1$, then $(a - b, a + b)$ is either 1 or 2.

2) [14 points] Remember that

$$(x^n - y^n) = (x - y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + \cdots + x^2y^{n-3} + xy^{n-2} + y^{n-1}).$$

Prove that if $a^n - 1$ is prime for some $a, n \in \mathbb{Z}_{>1}$, then $a = 2$ and n is itself prime.
[**Hint:** First, prove that $a = 2$. For the second part, assume that $n = r \cdot s$, with $r, s > 1$, to derive a contradiction. Note that $2^n = 2^{rs} = (2^r)^s$.]