April 8th, 2014

Math 421

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Spring 2014

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 2

You have one hour and fifteen minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) [20 points] How many permutations in S_{20} have a longest cycle [in its decomposition into disjoint cycles] of length 12?

2) [20 points] Six guests are to sit in any number of identical round tables. In how many ways can this be done so that no guest sits alone at a table.

3) [20 points] Compute

$$\sum_{k=0}^{30} (-1)^{k+1} c(30, k) 2^k.$$

4) [20 points] In how many permutations of S_{50} does 1 appear in a 11-cycle and 2 in 17-cycle [in its decomposition into disjoint cycles]?

5) [20 points] Let

$$p(n) = \left|\{\sigma \in S_n \ : \ \sigma^3 = \mathrm{id}\}\right|.$$

[Remember that $\sigma^3 = \mathrm{id}$ if and only if its cycle decomposition has only 1-cycles and/or 3-cycles.] Prove that if $p(n)$ satisfies:

$$p(n) = p(n-1) + (n-1)(n-2) \cdot p(n-3).$$

Scratch: