February 25th, 2014

Math 421

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Spring 2014

Name: ....

Student ID (last 6 digits): XXX-................

Midterm 1

You have one hour and fifteen minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 8 questions and 10 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	10	
2	10	
3	15	
4	10	
5	10	
6	15	
7	15	
8	15	
Total	100	

1) [10 points] In how many ways can one choose a dozen doughnuts from five different kinds?

2) [10 points] Prove that for any positive integer n we have

$$3^n = \sum_{k=0}^{n} \binom{n}{k} 2^k.$$

3) [15 points] A child wants to give away three gift bags made by choosing their content form seven different toys. [No bag can be empty, and each toy *must* go in only one bag.] How many different configurations are possible? Here the answer must be a *number*, with no computations left indicated.

4) [10 points] A host invites n couples to a party. She wants to invite k people, with $k \leq n$, from the $2n$ guests to give speeches, but she does *not* want to invite both members of any couple. In how many ways can she invite the speakers?

5) [10 points] Prove that the number of partitions of n in *exactly* k parts is the same as the number of partitions of n in which the largest part is k.

6) [15 points] Prove that for any positive integer n we have

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2.$$

7) [15 points] Find the number of compositions of 14 into [any number of] *even* parts. Here the answer must be a *number*, with no computations left indicated.

8) [15 points] In how many ways can the elements of $\{1, 2, \ldots, n\}$ be ordered in such a way that the sum of any two consecutive elements is *odd*?
[**Hint:** A sum of two integers is odd if and only if one of them is even and the other is odd. By the way, does it make a difference if n itself is even or odd?]

Scratch: