April 2nd, 2013

Math 307

Luís Finotti
Spring 2013

Name:...

Student ID (last 6 digits): XXX- ....

Midterm 2

You have 75 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 6 questions and 9 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**. Remember you also be graded on how well written your proofs are.

Good luck!

Question	Max. Points	Score
1	15	
2	15	
3	15	
4	15	
5	25	
6	15	
Total	100	

1) [15 points] Suppose that A, B and C are sets such that $A \subseteq B \setminus C$. Show that $A \cap C = \varnothing$.

2) [15 points] Let B be a set and $\mathcal{F} \subseteq \mathscr{P}(B)$. Show that $\bigcup \mathcal{F} \subseteq B$.

3) [15 points] Let x be an integer. Show that there exists an integer k such that either $x^2 = 4k$ or $x^2 = 4k + 1$. [**Hint:** Remember that x is even if there is an integer n such that $x = 2n$ and it is odd if there is an integer n such that $x = 2n + 1$.]

4) [15 points] Let A, B, C and D be *non-empty* sets. Prove that if $A \times B \subseteq C \times D$ then $A \subseteq C$ and $B \subseteq D$.

5) Let R be the relation on $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$:

$$R = \{(a, b) \in \mathbb{N} \times \mathbb{N} \ : \ b = 3a\}.$$

(a) [3 points] What is the domain of R? No need to justify.

(b) [3 points] What is the range of R? No need to justify.

(c) [3 points] What is the range of $R \circ R$? No need to justify.

(d) [3 points] Give three different elements of R^{-1}. No need to justify.

(e) [13 points] Check if R is each reflexive, symmetric and transitive? Justify each answer with proofs [in the affirmative case] or counter-examples [in the negative case].

6) [15 points] Suppose that R_1 and R_2 are symmetric relations on A. Prove that $R_1 \cup R_2$ is also symmetric.

Scratch: