May 3rd, 2011

Math 141

Name: .....

Luís Finotti
Spring 2011

Student ID (last 6 digits): XXX- .....

TA recitation (check one):

Cody Lorton:	☐ 8:00	Craig Collins:	☐ 11:15
	☐ 9:05		☐ 12:25

Final

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 9 questions and 13 printed pages (including this one and two pages for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	5	
2	20	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	15	
Total	100	

1) [5 points] Let $f(x) = \dfrac{(x-1)(x+2)(x^2-4x+3)}{x}$. Find where $f(x) = 0$, $f(x) > 0$, and $f(x) < 0$.

2) Compute the following limits.

(a) [5 points] $\lim_{x \to 1^+} \dfrac{x^3 + 2x - 4}{x^2 - x}$

(b) [7 points] $\lim_{x \to 0} \dfrac{\mathrm{e}^x - x - 1}{x^2}$.

(c) [8 points] $\lim_{x \to \infty} x^2 \sin\left(\frac{1}{4x^2}\right)$.

3) [10 points] If $f(x) = \cos(x)^{x/(e^x+1)}$, compute the derivative $f'(x)$. *You do not need to simplify!*

4) [10 points] Consider the curve given by the equation $x^3 + y^4 - y - 1 = 0$ and the curve given by the parametric equations $x = (t+1)\mathrm{e}^t$, $y = \arcsin(t^2) + 1$. Are the tangent lines at the point $(1, 1)$ [which is indeed a point in both curves] orthogonal?

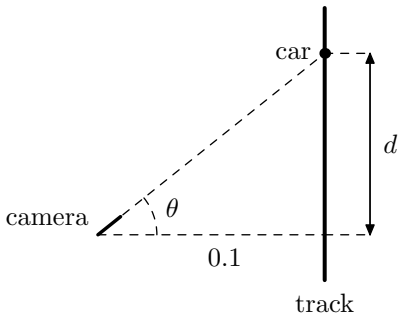
[**Hint:** The second curves passes through $(1, 1)$ when $t = 0$. Also, if you cannot find the tangent lines, you can describe how you'd find if they are perpendicular for a *little* partial credit.]

5) [10 points] A particle moves along a straight line with position [measured as the distance to a fixed point] at time t given by $s(t) = t^4/2 + t^3 - 6t^2$. [Units can be taken to be meters for distance and seconds for time.] For $t \in [0, 4]$ only, when was the velocity of the particle maximal and when was it minimal?

[**Note:** A negative velocity means that the particle is moving backwards. We do consider a negative velocity to be smaller than any positive velocity.]

6) [10 points] You want to build a box of volume $10\,\text{ft}^3$ and with a *square bottom*. The cost for the material to build the bottom, sides, and top cost \$4, \$2, and \$1 per square foot respectively. Find the dimensions of the box of minimal cost, *as well as* the cost to build such box.

7) [10 points] A fixed position [but rotating] camera is placed 0.1 meter away from a straight race track and it is following a race car which is moving at speed of 50 meters per second. [See picture below. You can assume that the car is moving "up" in the picture.] At what speed is the camera rotating [with units radians per second] when it is facing the car with an angle with respect to the line to the closest point of the track [denoted by θ in the figure] of $\pi/4$ radians?

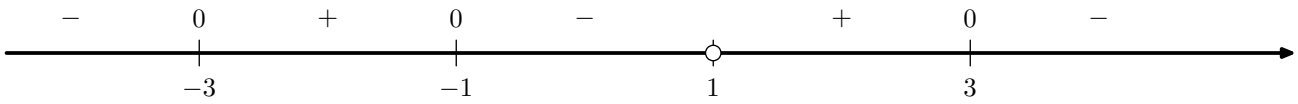


8) [10 points] Use differentials/linear approximation to estimate the amount of paint [in cubic centimeters] needed to apply a coat of paint 0.05 cm thick to a sphere of diameter 20 cm.

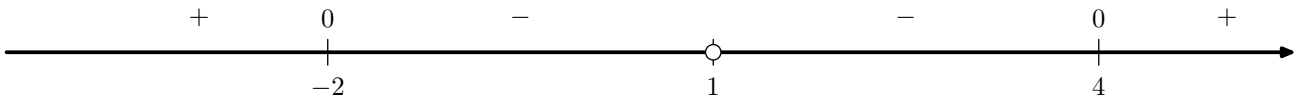
[**Hint:** The volume of a sphere is $V = 4\pi r^3/3$.]

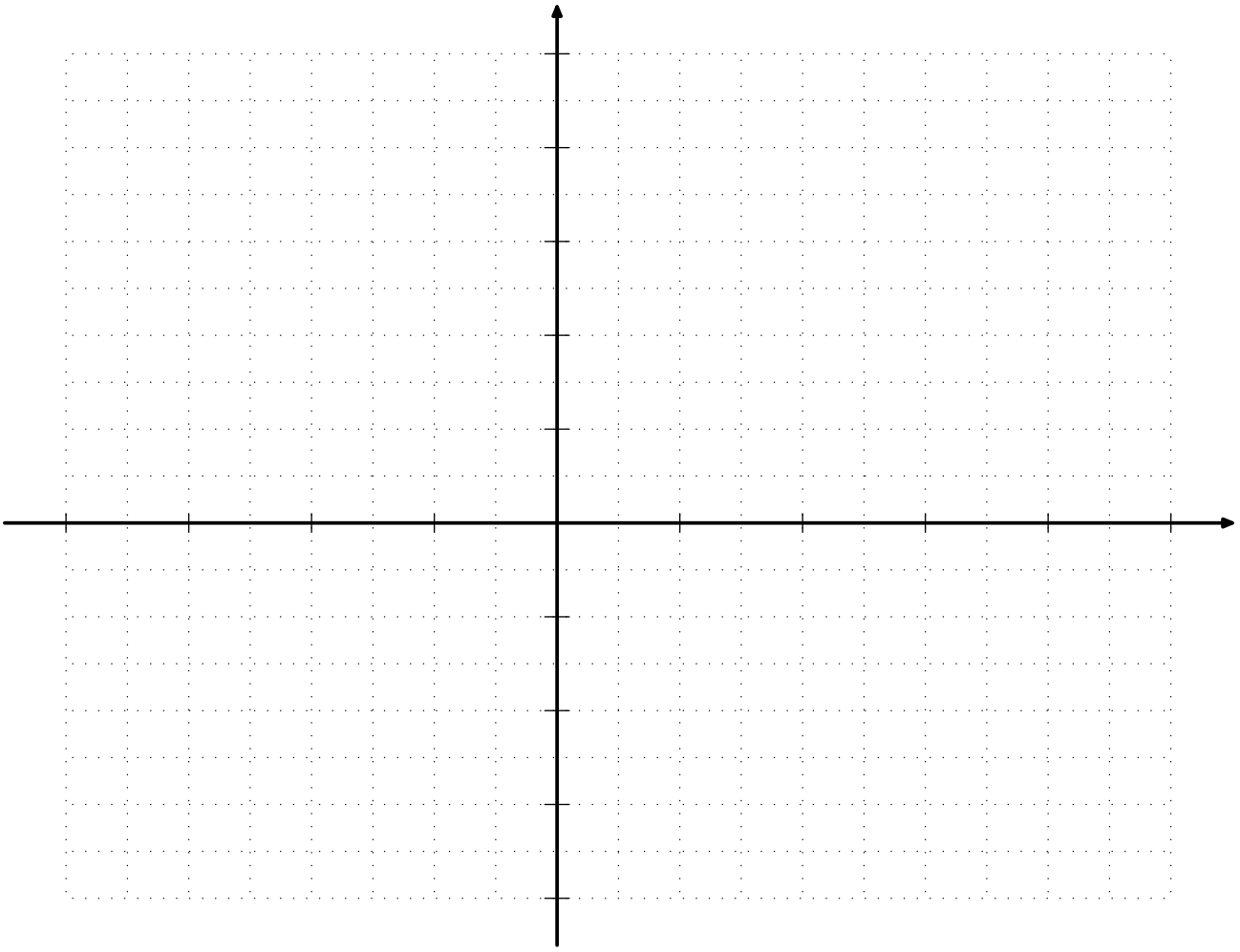
9) [15 points] Sketch the graph of a function $f(x)$ which satisfies all of the following conditions [draw concavities carefully!]:

- domain is all real numbers except 1;
- x-intercepts are -3, 0.25, 1.5, and y-intercept is 1.5;
- $f(-2) = 1.5$, $f(-1) = 3$, $f(3) = 3$, $f(4) = 1.5$;
- $\lim_{x\to-\infty} f(x) = \infty$, $\lim_{x\to\infty} f(x) = 0$, $\lim_{x\to 1} f(x) = -\infty$;
- the sign of the derivative is given by:



- the sign of the second derivative is given by:





Scratch: