

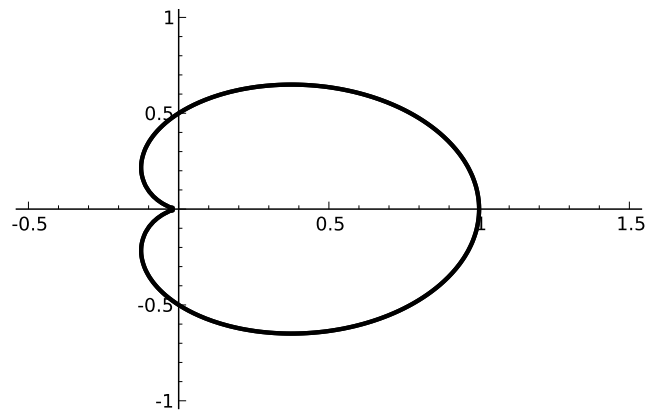
1) Given $f(x)$, compute the derivatives $f'(x)$.

(a) [6 points] $f(x) = \left(\frac{e^{2x}}{x^2 + 1} \right)^5$

(b) [7 points] $f(x) = \cos(2^x) \cdot \arctan(\sqrt{x})$

(c) [7 points] $f(x) = x^{\ln(x)}$

2) [20 points] The equation $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ gives a *cardioid*. [See the picture below.] Find equation of the tangent line at $(0, 1/2)$.

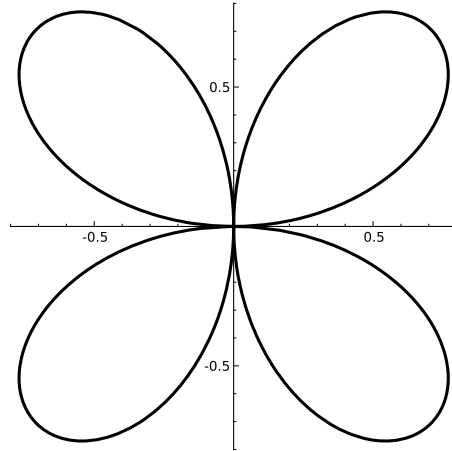


3) [20 points] Consider the parametrized curve [pictured below] given by

$$x = \cos(t) \sin(2t),$$

$$y = \sin(t) \sin(2t).$$

Show that the tangent lines for $t = \pi/4$ and $t = -\pi/4$ are perpendicular.



4) [20 points] The circumference of a sphere [i.e., the length of the “equator” of the sphere] was measured to be 8 cm, with possible error of 0.5 cm. Estimate the maximal error *and* the relative error that can occur with the volume of the sphere.

[Hint: If r is the radius of a sphere, its circumference C and volume V are given by $C = 2\pi r$ and $V = 4\pi r^3/3$ respectively. *You might need to write V in terms of C [instead of r].*

5) [20 points] A 10 ft long ladder rests against a [vertical] wall. If the bottom of the ladder slides away from wall at a rate of 1 ft/s, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

Scratch: