## Math 141

Luís Finotti

Spring 2011
Name:
Student ID (last 6 digits): XXX-

TA recitation (check one):
Cody Lorton: $\square$ 8:00 Craig Collins: $\square 11: 15$12:25

## Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 8 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive zero. Also, points will be taken from messy solutions.

## Good luck!

| Question | Max. Points | Score |
| :---: | :---: | :---: |
| 1 | 20 |  |
| 2 | 20 |  |
| 3 | 20 |  |
| 4 | 20 |  |
| 5 | 20 |  |
| Total | 100 |  |

1) Given $f(x)$, compute the derivatives $f^{\prime}(x)$.
(a) $[6$ points $] f(x)=\left(\frac{\mathrm{e}^{2 x}}{x^{2}+1}\right)^{5}$
(b) [7 points] $f(x)=\cos \left(2^{x}\right) \cdot \arctan (\sqrt{x})$
(c) [7 points] $f(x)=x^{\ln (x)}$
2) [20 points] The equation $x^{2}+y^{2}=\left(2 x^{2}+2 y^{2}-x\right)^{2}$ gives a cardioid. [See the picture below.] Find equation of the tangent line at $(0,1 / 2)$.

3) [20 points] Consider the parametrized curve [pictured below] given by

$$
\begin{aligned}
& x=\cos (t) \sin (2 t), \\
& y=\sin (t) \sin (2 t) .
\end{aligned}
$$

Show that the tangent lines for $t=\pi / 4$ and $t=-\pi / 4$ are perpendicular.

4) [20 points] The circumference of a sphere [i.e., the length of the "equator" of the sphere] was measured to be 8 cm , with possible error of 0.5 cm . Estimate the maximal error and the relative error that can occur with the volume of the sphere.
[Hint: If $r$ is the radius of a sphere, its circumference $C$ and volume $V$ are given by $C=2 \pi r$ and $V=4 \pi r^{3} / 3$ respectively. You might need to write $V$ in terms of $C$ [instead of $r$ ].]
5) [20 points] A 10 ft long ladder rests against a [vertical] wall. If the bottom of the ladder slides away from wall at a rate of $1 \mathrm{ft} / \mathrm{s}$, how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 6 ft from the wall?

## Scratch:

