

# Final (Take Home)

M552 – Abstract Algebra

May 6th, 2008

- You are not supposed to discuss this with anyone.
  - You can use Dummit and Foote, Lang, and class notes, but please do not keep looking for solutions (in several books, papers, internet, etc.).
  - Please, since you have some time, write your solutions neatly.
  - The due date is Tuesday, 05/06 before the final.
  - If you turn it in early, please take it to me (in my office). If I'm not there, please slide it under the door. DO NOT LEAVE IT IN MY MAILBOX. (If you type it, you can just e-mail it to me too.)
  - *Please give references to results that you use!* [Well, unless it's trivial or something we've used many times.]
1. [20 points] Let  $K/F$  be an algebraic extension such that every non-constant  $f(x) \in F[x]$  has at least one root in  $K$ . Prove that  $K$  is algebraically closed [and hence  $K$  is the algebraic closure of  $F$ ].  
[**Hint:** Be careful with inseparability. Also, the *Primitive Element Theorem* might be useful.]
  2. [20 points] Let  $K/F$  be an algebraic, but *infinite* extension. Let  $G \stackrel{\text{def}}{=} \text{Aut}(K/F)$ . Prove that  $G$  is *residually finite*, i.e., that the intersection of all subgroups of  $G$  which are normal and of finite index [in  $G$ ] is  $\{1\}$ .