Midterm 4

Math 300 – Fall 2020

November 11th, 2020

Instructions

- *Write neatly and legibly!*
- Your camera *must* be on at *all times* and showing you properly. (You can only leave Zoom when you are done!)
- Leave the sound on (not the mic), so that you can *hear* incoming private messages or if I need to say something to all.
- You do not need to copy the statements. Just number your answers.
- Each problem must be solved in a different page, but items of the same problem can be in the same page.
- If you have any questions, send me a private message through the chat.
- You can only use your computer to look at the exam or to send me a message.
- **When you are done with the exam and are about to start scanning/uploading, send me a private message!** (Something like *"Scanning now."*)
- Make sure your scans are legible before uploading them to Canvas.
- **When you are done uploading your solutions, send me a private message.** (Something like *"Done."* No need for the time.) You can then leave Zoom.
- **Be prepared to, upon request (via private message), show me your surroundings!**

1) Suppose that R is a relation from A to B and S and is a relation from B to C. Prove that if $\text{Ran}(R) \subseteq \text{Dom}(S)$, then $\text{Dom}(R) \subseteq \text{Dom}(S \circ R)$.
[**Note:** This was (part of) a HW problem.]

2) Let R_1 and R_2 be relations on a set A.

(a) Prove that if R_1 and R_2 are both reflexive, then so is $R_1 \cup R_2$.

(b) Prove that if R_1 and R_2 are both symmetric, then so is $R_1 \cup R_2$.

3) Let R be a partial order on A, $B_1 \subseteq A$, $B_2 \subseteq A$, x_1 a least upper bound of B_1, and x_2 an upper bound of B_2. Prove that if $B_1 \subseteq B_2$, then $x_1 R x_2$.
[**Note:** This was a HW problem.]

4) Let m be a [fixed] positive integer and consider the relation on $\mathbb{Z}$ given by:

$$C_m = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid m \text{ divides } b - a\}.$$

Prove that C_m is an equivalence relation.
[Remember that m divides c iff $c = m \cdot k$ for some k in $\mathbb{Z}$.]

5) Let $f : A \to B$, $C \subseteq A$, and $g = f \cap (C \times B)$. Prove that $g : C \to B$ (i.e., the relation g is a function from C to B) and for all $c \in C$ we have that $g(c) = f(c)$.