Midterm 2

Math 300 – Fall 2020

September 30th, 2020

Instructions

- *Write neatly and legibly!*

- Your camera *must* be on at *all times* and showing you properly. (You can only leave Zoom when you are done!)

- Leave the sound on (not the mic), so that you can *hear* incoming private messages or if I need to say something to all.

- You do not need to copy the statements. Just number your answers.

- Each problem must be solved in a different page, but items of the same problem can be in the same page.

- If you have any questions, send me a private message through the chat.

- You can only use your computer to look at the exam or to send me a message.

- **When you are done with the exam and are about to start scanning/uploading, send me a private message!** (Something like *"Scanning now."*)

- Make sure your scans are legible before uploading them to Canvas.

- **When you are done uploading your solutions, send me a private message.** (Something like *"Done."* No need for the time.) You can then leave Zoom.

- **Be prepared to, upon request (via private message), show me your surroundings!**

1) Analyze the logical form of the following statements:

(a) Anyone who has bought a Rolls Royce must have a rich uncle.

Use the statements

- $B(x)$ for "x bought a Rolls Royce",
- $U(x, y)$ for "x is y's uncle", and
- $R(x)$ for "x is rich".

(b) "If anyone in the dorm has the measles, then everyone who has a friend in the dorm will have to be quarantined."

Use:

- $M(x)$ for "x has measles",
- $F(x, y)$ for "x and y are friends",
- $Q(x)$ for "x will have to be quarantined", and
- D for the *set* of everyone living in the dorm.

Your quantifiers *cannot* be bound! If can only do it with bound quantifiers and you bound them correctly, you *do* get some partial credit.

2) Negate the following statement and restate it as a positive statement:

$$\forall e \in \mathbb{R}_{>0} \left[\exists d \in \mathbb{R}_{>0} \left(\forall x (x < d \rightarrow x^2 < e) \right) \right].$$

3) Verify the equality

$$\bigcap_{i \in I} (A_i \setminus B_i) = \left(\bigcap_{i \in I} A_i \right) \setminus \left(\bigcup_{i \in I} B_i \right)$$

by showing (with logical symbols) that

$$x \in \bigcap_{i \in I} (A_i \setminus B_i) \sim x \in \left(\bigcap_{i \in I} A_i \right) \setminus \left(\bigcup_{i \in I} B_i \right).$$

[**Note:** This was a HW problem.]

Continues in the next page!

4) Suppose that $A \setminus B \subseteq C \cap D$ and $x \in A$. Prove that if $x \notin D$, then $x \in B$. [This has to be proved "Chapter 3 style": in proper English, and not just with symbols.]

5) Suppose that $y + x = 2y - x$, and that x and y are not both zero. Prove that $y \neq 0$. [This has to be proved "Chapter 3 style": in proper English, and not just with symbols.] [**Note:** This was a HW problem.]