1) Fill in the truth-table below. (Copy to your solution and fill it out. You can add extra columns if you want, but you need to have the three columns below filled. You can also change the order of rows, if you prefer.)

P	Q	R	$P \wedge \neg Q$	$Q \to \neg R$	$(P \wedge \neg Q) \vee (Q \to \neg R)$
F	F	F	F	T	T
F	F	T	F	T	T
F	T	F	F	T	T
F	T	T	F	F	F
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	F	T	T
T	T	T	F	F	F

2) Analyze the logical form of the following statements. (Do not forget to use parentheses when necessary!)

(a) "You have to be American to be the president, but being American doesn't make you the president." (Use the statements A for "you are American" and P for "you are the president".)

Solution.

$$(P \to A) \wedge \neg(A \to P)$$

□

(b) "The only way to pass an exam is to either study very hard or be very lucky." (Use the statements P for "you pass the exam", S for "you study very hard", and L for "you are very lucky".)

Solution.

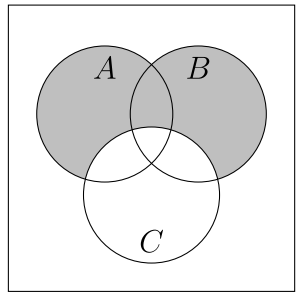
$$P \to (S \vee L).$$

□

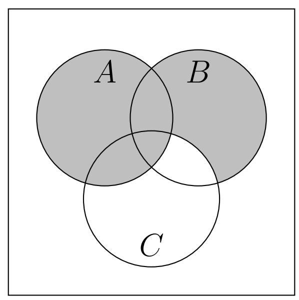
3) Consider the sets $(A \cup B) \setminus C$ and $A \cup (B \setminus C)$.

(a) Draw the Venn diagrams for both sets.

Solution. We have that $(A \cup B) \setminus C$:



And $A \cup (B \setminus C)$:



□

(b) Give simple concrete examples for the sets A, B, and C for which we have $(A \cup B) \setminus C \neq A \cup (B \setminus C)$. (Make sure to show why they are different by computing both resulting sets.)

Solution. Let $A = B = C = \{1\}$. Then,

$$(A \cup B) \setminus C = (\{1\} \cup \{1\}) \setminus \{1\} = \{1\} \setminus \{1\} = \varnothing,$$

while

$$A \cup (B \setminus C) = \{1\} \cup (\{1\} \setminus \{1\}) = \{1\} \cup \varnothing = \{1\}.$$

□

4) Use the laws of formal logic to simplify the statement

$$(P \wedge R) \vee [\neg R \wedge (P \vee Q)].$$

You don't have to name the rules you use, but use one rule per step. (Do not skip steps!)
[This was a HW problem. **Hint:** It should simplify to $P \vee (\neg R \wedge Q)$.]

Solution.

$$
\begin{aligned}
(P \wedge R) \vee [\neg R \wedge (P \vee Q)] &\sim (P \wedge R) \vee [(\neg R \wedge P) \vee (\neg R \wedge Q)] && \text{[Distr. Law]} \\
&\sim [(P \wedge R) \vee (\neg R \wedge P)] \vee (\neg R \wedge Q) && \text{[Assoc. Law]} \\
&\sim [(P \wedge R) \vee (P \wedge \neg R)] \vee (\neg R \wedge Q) && \text{[Comm. Law]} \\
&\sim [P \wedge (R \vee \neg R)] \vee (\neg R \wedge Q) && \text{[Distr. Law]} \\
&\sim [P \wedge \text{taut.}] \vee (\neg R \wedge Q) && \\
&\sim P \vee (\neg R \wedge Q) && \text{[Tautology]}
\end{aligned}
$$

□

5) Prove that $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$ by showing that $x \in (A \cup B) \setminus C$ is logically equivalent to $x \in (A \setminus C) \cup (B \setminus C)$.
[This was a HW problem.]

Solution.

$$
\begin{aligned}
x \in (A \cup B) \setminus C &\sim [x \in (A \cup B)] \wedge \neg[x \in C] \\
&\sim [(x \in A) \vee (x \in B)] \wedge \neg[x \in C] \\
&\sim [(x \in A) \wedge \neg(x \in C)] \vee [(x \in B) \wedge \neg(x \in C)] \\
&\sim (x \in A \setminus C) \vee (x \in B \setminus C) \\
&\sim x \in (A \setminus C) \cup (B \setminus C)
\end{aligned}
$$

□