Midterm 1

Math 300 – Fall 2020

September 9th, 2020

Instructions

- *Write neatly and legibly!*

- Your camera *must* be on at *all times* and showing you properly. (You can only leave Zoom when you are done!)

- Leave the sound on (not the mic), so that you can *hear* incoming private messages or if I need to say something to all.

- You do not need to copy the statements. Just number your answers.

- Each problem must be solved in a different page, but items of the same problem can be in the same page.

- If you have any questions, send me a private message through the chat.

- You can only use your computer to look at the exam or to send me a message.

- **When you are done with the exam and are about to start scanning/uploading, send me a private message!** (Something like *"Scanning now."*)

- Make sure your scans are legible before uploading them to Canvas.

- **When you are done uploading your solutions, send me a private message.** (Something like *"Done."* No need for the time.) You can then leave Zoom.

- **Be prepared to, upon request (via private message), show me your surroundings!**

1) Fill in the truth-table below. (Copy to your solution and fill it out. You can add extra columns if you want, but you need to have the three columns below filled. You can also change the order of rows, if you prefer.)

P	Q	R	$P \wedge \neg Q$	$Q \rightarrow \neg R$	$(P \wedge \neg Q) \vee (Q \rightarrow \neg R)$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

2) Analyze the logical form of the following statements:

(a) "You have to be American to be the president, but being American doesn't make you the president." (Use the statements A for "you are American" and P for "you are the president".)

(b) "The only way to pass an exam is to either study very hard or be very lucky." (Use the statements P for "you pass the exam", S for "you study very hard", and L for "you are very lucky".)

3) Consider the sets $(A \cup B) \setminus C$ and $A \cup (B \setminus C)$.

(a) Draw the Venn diagrams for both sets.

(b) Give simple concrete examples for the sets A, B, and C for which we have $(A \cup B) \setminus C \neq A \cup (B \setminus C)$. (Make sure to show why they are different by computing both resulting sets.)

4) Use the laws of formal logic to simplify the statement

$$(P \wedge R) \vee [\neg R \wedge (P \vee Q)].$$

You don't have to name the rules you use, but use one rule per step. (Do not skip steps!)
[This was a HW problem. **Hint:** It should simplify to $P \vee (\neg R \wedge Q)$.]

5) Prove that $(A \cup B) \setminus C = (A \setminus C) \cup (B \setminus C)$ by showing that $x \in (A \cup B) \setminus C$ is logically equivalent to $x \in (A \setminus C) \cup (B \setminus C)$.
[This was a HW problem.]