December 8th, 2017

Math 351

Luís Finotti
Fall 2017

Name:...

Student ID (last 6 digits): XXX- ....

Final

Question	Max. Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	20	
8	10	
9	10	
Total	100	

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 9 questions and 13 printed pages (including this one and two pages for scratch work in the end).

No books or notes are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

1) [10 points] Find all integers x such that

$$\begin{aligned} x &\equiv 2 \pmod{9}, \\ x &\equiv 4 \pmod{11}. \end{aligned}$$

[Of course, x must satisfy *both* congruences.]

2) [10 points] Let $a, b \in \mathbb{Z} \setminus \{0\}$ and $d = (a, b)$. Prove that $(a/d, b/d) = 1$.

[**Note:** This was an old HW problem.]

3) [10 points] Prove that if $x, y, z \in \mathbb{Z}$, none divisible by 3, then $x^2 + y^2 \neq z^2$.

4) [10 points] Let R be a *domain*. Prove that if $f \in U(R[x])$, then $f \in U(R)$ [i.e., f is a constant polynomial and a unit of R].

[**Note:** This was proved in class.]

5) The statements below are *false*. Give a counter example to each one.

(a) [3 points] If F is a field, and $a \in F$, then $a = -a$ only if $a = 0$.

(b) [3 points] If R is a ring and $f \in R[x]$, then $\deg(f^2) = 2\deg(f)$.

(c) [4 points] If R is a ring and $f \in R[x]$ with $\deg(f) = n$, then f has at most n roots in R.

6) Determine if the polynomials below are irreducible or not in the corresponding polynomial ring. *Justify each answer!*

(a) [3 points] $f = x^5 - 4x^4 + 10x^3 + 8x^2 - 2x + 6$ in $\mathbb{Q}[x]$.

(b) [4 points] $f = 6666667x^3 - 33333334x + 99999991$ in $\mathbb{Q}[x]$.

(c) [3 points] $f = 3x^4 - 6x^3 + 9x - 1$ in $\mathbb{Q}[x]$.

7) Let $\sigma, \tau \in S_9$ be given by

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 3 & 9 & 2 & 1 & 4 & 8 & 7 & 6 \end{pmatrix} \quad \text{and} \quad \tau = (1\,3\,7\,8)(2\,4\,5\,9).$$

(a) [4 points] Write the *complete* factorization of σ into disjoint cycles.

(b) [3 points] Compute $\tau\sigma$. [Your answer can be in any form.]

(c) [3 points] Write τ as a product of transpositions.

Continues on next page.

(d) [4 points] Compute $\sigma\tau\sigma^{-1}$. [Your answer can be in any form.]

(e) [3 points] Compute $\mathrm{sign}(\tau)$.

(f) [3 points] Compute $|\tau|$.

8) [10 points] Let G be a group, H and K finite subgroups of G such that $(|H|, |K|) = 1$. Prove that $H \cap K = \{1\}$.

[**Note:** This was a HW problem.]

9) [10 points] Let G be a group and suppose that $(ab)^3 = 1$ for some $a, b \in G$. Prove that $(ba)^3 = 1$.

[**Note:** There is nothing special about the exponent 3. After you do this, you should see how to do it for any exponent.]

Scratch:

Scratch: