November 20th, 2017

Math 351

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Fall 2017

Name: .....

Student ID (last 6 digits): XXX- .....

Midterm 4

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 9 printed pages (including this one and a page for scratch work at the end).

No calculators, books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	40	
Total	100	

1) [20 points] If

$$f = 2 \cdot (x+2)^3 \cdot (x^2+2)^2 \cdot (x^2+3x+3) \cdot (x^3+x+1)^5,$$
$$g = 3 \cdot (x+1)^5 \cdot (x^2+2) \cdot (x^2+3x+3)^3,$$

are the factorizations of f an g into monic irreducible polynomials in $\mathbb{F}_5[x]$, then give the factorization of their GCD and LCM.

2) [20 points] Let $f(x) = x^4 + x^2 + x + 1$ and $g(x) = x^3 + x^2 + x + 1$, both in $\mathbb{F}_2[x]$. Express their GCD as a linear combination of themselves.
[**Hint:** You should find that the GCD is $x + 1$.]

3) [20 points] Let F be a field and $f, g, h \in F[x]$ with f and g relatively prime. Prove that if $f \mid h$ and $g \mid h$, then $(f \cdot g) \mid h$.

[**Hint:** If you could prove it in $\mathbb{Z}$ instead of $F[x]$, the same proof should work here. Also, this was a HW problem.]

4) [40 points] Decide if the polynomials below are irreducible or not in the corresponding polynomial ring. [Justify!]

(a) $f = x^2 - 3x + 5$ in $\mathbb{R}[x]$.

(b) $f = x^5 - x + 2$ in $\mathbb{C}[x]$.

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(c) $f = \frac{2}{3}x^3 + 4x^2 - 6x + \frac{4}{3}$ in $\mathbb{Q}[x]$.

(d) $f = 3x^5 - 9x^4 + 6x^2 + 12x - 3335$ in $\mathbb{Q}[x]$.

Continues on next page!

(e) $f = x + 1000$ in $\mathbb{F}_{2017}[x]$.

(f) $f = 1000x^3 - 999x^2 - 1001x + 20000$ in $\mathbb{Q}[x]$.

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(g) $f = 3x^7 - 4x^6 + 18x^5 + 6x^4 + 2x^3 - 34x^2 + 100x - 30$ in $\mathbb{Q}[x]$.

(h) $f = 2x^9 + 5x^7 + 3x^5 + x^4 + 6x^3 + 4x$ in $\mathbb{F}_7[x]$.

Scratch: