November 1st, 2017

Math 351

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Fall 2017

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 3

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 8 printed pages (including this one, a page for scratch work, and a page with ring axioms in the end).

No calculators, books or notes are allowed on this exam, but you can use your own index cards!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	30	
3	30	
4	20	
Total	100	

1) [20 points] If u is a unit in a *commutative* ring, prove that it's inverse is unique: if $ua = 1$ and $ub = 1$, then $a = b$. *Justify every step with an axiom! (Don't skip steps!)* [The axioms are listed in the last page.]

2) Prove or disprove [i.e., if the statement is true, prove it, if not, show why the statement is false].

(a) [15 points] $R = \{f \in \mathbb{Z}[x] \ : \ f \text{ is monic}\}$ is a domain.

(b) [15 points] $R = \{a + x^2 f \ : \ a \in \mathbb{Z} \text{ and } f \in \mathbb{Z}[x]\}$ is a domain.

3) Examples of rings (no justifications needed):

(a) [15 points] Give an example of an infinite, *non-commutative* ring R such that $2 \cdot a = 0$ for all $a \in R$.

(b) [15 points] Give an example of a ring R that is not a field, but *contains* an *infinite* field and such that $25 \cdot a = 0$ for all $a \in R$.

4) [20 points] Prove that if $f = x^p - x \in \mathbb{F}_p[x]$, then $f(a) = 0$ for all $a \in \mathbb{F}_p$.

Scratch:

Commutative Ring Axioms: A [non-empty] set with two operations, + and $\cdot$, is a commutative ring if:

0. For all $a, b \in R$ we have that $a + b \in R$ and $a \cdot b \in R$.
1. For all $a, b \in R$ we have that $a + b = b + a$.
2. For all $a, b, c \in R$ we have that $(a + b) + c = a + (b + c)$.
3. There exists $0 \in R$ such that for all $a \in R$ we have $a + 0 = a$.
4. For all $a \in R$ there exists $-a \in R$ such that $a + (-a) = 0$.
5. For all $a, b \in R$ we have that $a \cdot b = b \cdot a$.
6. For all $a, b, c \in R$ we have that $(a \cdot b) \cdot c = a \cdot (b \cdot c)$
7. There is $1 \in R$ such that for all $a \in R$ we have that $1 \cdot a = a$
8. For all $a, b, c \in R$ we have that $a \cdot (b + c) = a \cdot b + a \cdot c$