November 21st, 2016

Math 307

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Fall 2016

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 5

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 6 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) [25 points] Let n be a [fixed] positive integer and define:

$$R = \{(a, b) \in \mathbb{Z} \times \mathbb{Z} \mid \; n \mid (a - b)\}.$$

Prove that R is an equivalence relation on $\mathbb{Z}$. [Remember, $x \mid y$ if there is $k \in \mathbb{Z}$ such that $y = x \cdot k$.]

[I mentioned this result in class and quickly said, but did not write, all the steps.]

Partial credit: If you can't do this or are stuck, I will give half credit [12 points] for the definitions of symmetric, reflexive and transitive relations.

2) [25 points] Let $f : A \to B$ and $g : B \to C$. Prove that if $g \circ f$ is one-to-one, then f is one-to-one.

[This was a homework problem.]

Partial credit: If you can't do this or are stuck, I will give some credit [10 points] for the definition of one-to-one.

3) [25 points] Let $f : A \to B$ and $g : B \to C$. Prove that if f is onto and g is not one-to-one, then $g \circ f$ is not one-to-one.
[This was a homework problem.]
Partial credit: If you can't do this or are stuck, I will give some credit [12 points] for the definition of onto and the *negation* of the definition of one-to-one.

4) [25 points] Let $f : A \to B$ and $g : B \to A$. Prove that if f is onto and $g \circ f = i_A$, then $f \circ g = i_B$. [Note that this means that $g = f^{-1}$.]
[This was done in a video.]

Scratch: