November 9th, 2016

Math 307

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Fall 2016

Name: . . . .

Student ID (last 6 digits): XXX- . . . .

Midterm 4

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	100	

1) Prove that if $A \times B$ and $C \times D$ are disjoint, then either A and C are disjoint or B and D are disjoint.
[This was a HW problem.]

2) [20 points] Let R be a relation from A to B and S be a relation from B to C. Prove that if $\text{Ran}(R) \subseteq \text{Dom}(S)$, then $\text{Dom}(R) \subseteq \text{Dom}(S \circ R)$.
[This was a *part of* a HW problem.]
Partial credit: If you can't do this or are stuck, I will give half credit [10 points] for the definitions for $b \in \text{Ran}(R)$, $b \in \text{Dom}(S)$ and $(a, c) \in S \circ R$.

3) [20 points] Let A be the set of all people and R be the relation such that for $a, b \in A$, we have that aRb iff a and b have at least one common parent. Answer the questions below. [If the answer is affirmative, explain. If not, *give a counterexample!*]

Partial credit: If you can't do this or are stuck, I will give half credit [10 points] for the definitions of the terms below.

(a) Is R reflexive?

(b) Is R symmetric?

(c) Is R transitive?

(d) Is R antisymmetric?

4) [20 points] Let $A = \mathscr{P}(\mathbb{N})$,

$$B = \{\{1\}, \{2\}, \{3\}, \{4\}, \{2,3\}, \{1,2,3\}, \{5,6,7,8,9\}\}$$

and consider the ordering on A given by [the usual] "$\subseteq$". [**No need to justify your answers here!**]

(a) List all minimal elements of B. [If none, just say so.]

(b) List all maximal elements of B. [If none, just say so.]

(c) Give the greatest lower bound for B. [If none, just say so.]

(d) Give the least upper bound for B. [If none, just say so.]

5) [20 points] Let R be an ordering relation on A and $B \subseteq A$. Prove that if there is $b \in B$ which is a lower bound for B, then it is also the smallest element of B *and* the greatest lower bound of B in A.

[So, there are two parts, but they are *really* simple and short!]

Partial credit: If you can't do this or are stuck, I will give half credit [10 points] for the definitions of smallest element, lower bound and greatest lower bound.

Scratch: