October 19th, 2016

Math 307

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Fall 2016

Name: ....

Student ID (last 6 digits): XXX- ....

Make Up Midterm 3

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 6 printed pages (including this one and a page for scratch work in the end).

No books or notes are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) [25 points] Let U be any set. Prove that there is a unique $A \in \mathscr{P}(U)$ such that for all $B \in \mathscr{P}(U)$, we have $A \cup B = B$.
[This is very similar to the HW problem on the last test, but we had $A \cap B = B$ instead of $A \cup B = B$. This current one was also done as an example in class and on a video.]

2) [25 points] Prove that if x is an integer *not* divisible by 3, then $x^2 + 3x - 1$ *is* divisible by 3.

[**Hint:** If an integer n is *not* divisible by 3, then its remainder when divided by 3 is either 1 or 2. So, in other words, n is *not divisible* by 3 iff either $n = 3k + 1$ or $n = 3k + 2$ for some $k \in \mathbb{Z}$.]

3) [25 points] Suppose the $I \neq \varnothing$ is a set of indices and let $\{A_i \mid i \in I\}$ and $\{B_i \mid i \in I\}$ be indexed families of sets. Prove that

$$\bigcup_{i \in I}(A_i \cap B_i) \subseteq \left(\bigcup_{i \in I} A_i\right) \cap \left(\bigcup_{i \in I} B_i\right).$$

[This is a HW problem, similar to the problem in the last exam.]

4) [25 points] Suppose the m and n are integers. Prove that if $m \cdot n$ is even, then either m or n is even. [This was an example from the book, also done in class.]

Scratch: