1) [15 points] Fill in the truth-table below:

P	Q	R	$\neg Q \wedge R$	$\neg P \vee R$	$[\neg Q \wedge R] \to [\neg P \vee R]$
T	T	T	F	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	F	T
F	T	T	F	T	T
F	T	F	F	T	T
F	F	T	T	T	T
F	F	F	F	T	T

2) [15 points] Use the rules of formal logic to simplify $\neg(\neg P \wedge Q) \vee (P \wedge \neg R)$. **In each step, state which rule you've used!**

[**Hint:** It should simplify to $P \vee \neg Q$.]

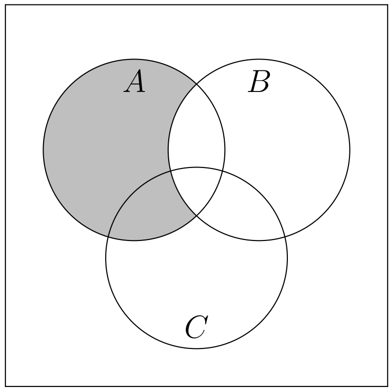
Solution. [This was a HW problem.]

$$
\begin{aligned}
\neg(\neg P \wedge Q) \vee (P \wedge \neg R) &\sim (\neg(\neg P) \vee \neg Q) \vee (P \wedge \neg R) && \text{[DeMorgan's Law]}\\
&\sim (P \vee \neg Q) \vee (P \wedge \neg R) && \text{[Double Negation]}\\
&\sim (\neg Q \vee P) \vee (P \wedge \neg R) && \text{[Commutativity]}\\
&\sim \neg Q \vee [P \vee (P \wedge \neg R)] && \text{[Associativity]}\\
&\sim \neg Q \vee P && \text{[Absorption]}\\
&\sim P \vee \neg Q && \text{[Commutativity].}
\end{aligned}
$$

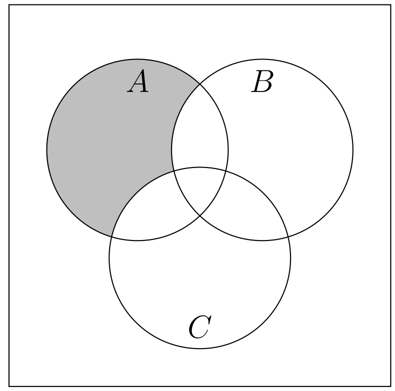
□

3) Sets and Venn Diagrams:

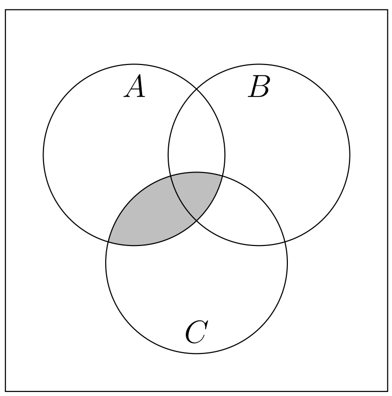
(a) [10 points] Fill in the following Venn Diagrams:



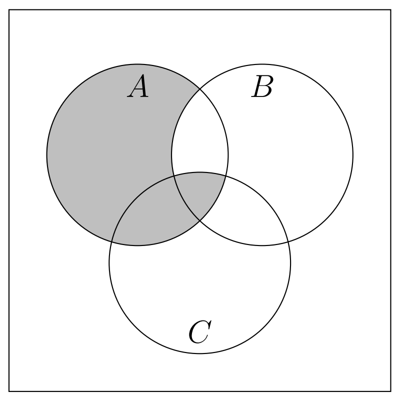
$A \setminus B$



$(A \setminus B) \setminus C$



$A \cap C$



$(A \setminus B) \cup (A \cap C)$

(b) [10 points] Give concrete examples of A, B and C for which

$$(A \setminus B) \setminus C \neq (A \setminus B) \cup (A \cap C).$$

Solution. Let $A = B = C = \{1\}$. Then, $A \setminus B = \varnothing$ and so $(A \setminus B) \setminus C = \varnothing$. On the other hand $(A \setminus B) \cup (A \cap C) = \varnothing \cup \{1\} = \{1\} \neq \varnothing$. So, the sets are different. $\square$

4) [15 points] Rewrite $\neg(P \leftrightarrow Q)$ using only $\wedge$, $\vee$ and $\neg$ [and P and Q, of course].

Solution. We have:

$$\begin{aligned} \neg(P \leftrightarrow Q) &\sim \neg[(P \to Q) \wedge (Q \to P)] \\ &\sim \neg(P \to Q) \vee \neg(Q \to P) \\ &\sim (P \wedge \neg Q) \vee (Q \wedge \neg P) \end{aligned}$$

□

5) [15 points] Show that $A \cup (B \setminus C) = (A \cup B) \setminus (C \setminus A)$ by showing that $x \in A \cup (B \setminus C)$ is [logically] equivalent to $x \in (A \cup B) \setminus (C \setminus A)$.

Solution. [This was a HW problem.]
We have:

$$\begin{aligned} x \in A \cup (B \setminus C) &\sim x \in A \vee x \in B \setminus C \\ &\sim x \in A \vee (x \in B \wedge \neg(x \in C)) \\ &\sim (x \in A \vee x \in B) \wedge (x \in A \vee \neg(x \in C)) \\ &\sim (x \in A \vee x \in B) \wedge \neg(\neg(x \in A) \wedge (x \in C)) \\ &\sim (x \in A \vee x \in B) \wedge \neg((x \in C) \wedge \neg(x \in A)) \\ &\sim (x \in A \cup B) \wedge \neg(x \in C \setminus A) \\ &\sim x \in (A \cup B) \setminus (C \setminus A). \end{aligned}$$

□

6) Analyze the logical form of the following statements.

(a) [10 points] You will pass this test and be happy as long as you've studied or are used to logic.

Solution. Let

$$P = \text{“you will pass the test”};$$
$$H = \text{“you will be happy”};$$
$$S = \text{“you've studied hard”};$$
$$L = \text{“you are used to logic”}.$$

Then, we have

$$(S \vee L) \to (P \wedge H).$$

□

(b) [10 points] Being relaxed but attentive is necessary, but not sufficient, to do well in exams.

Solution. Let

$$R = \text{“you are relaxed”};$$
$$A = \text{“you are attentive”};$$
$$E = \text{“you will do well on the exam”}.$$

Then, we have

$$[E \to (R \wedge A)] \wedge \neg[(R \wedge A) \to E].$$

□