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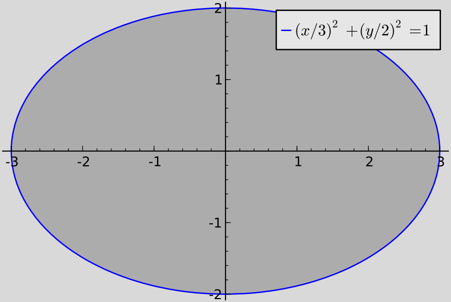

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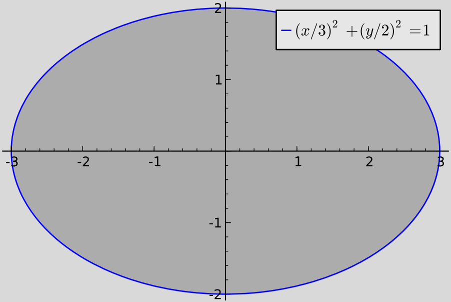


What's its area?

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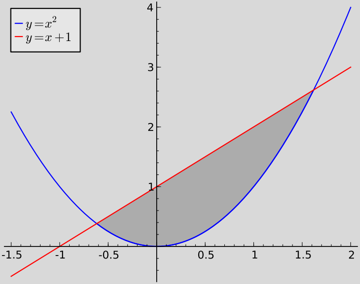

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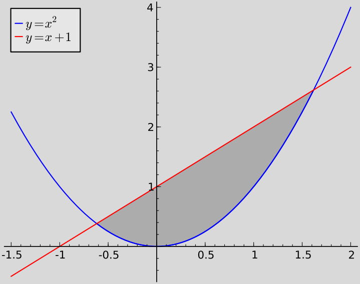

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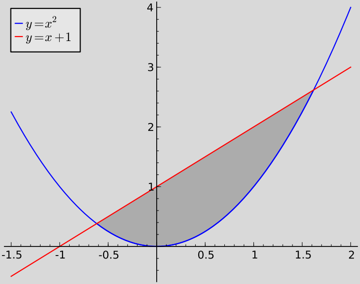


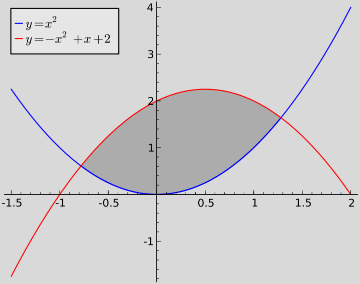

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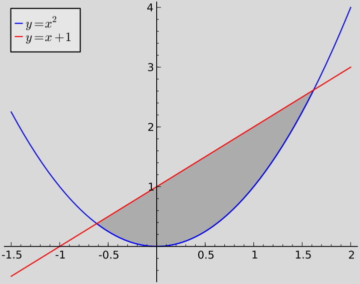


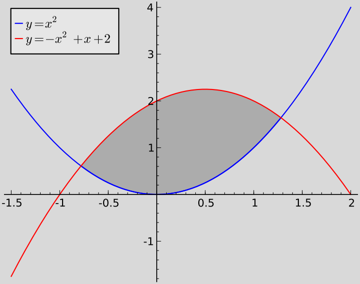


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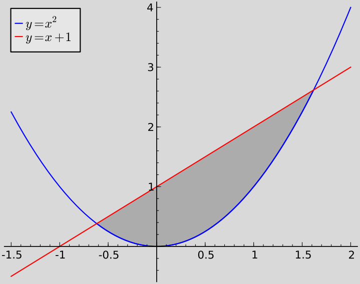


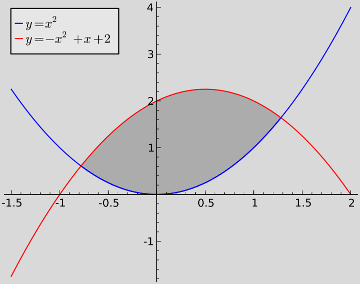


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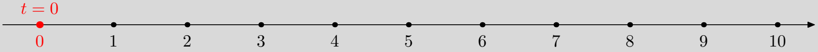

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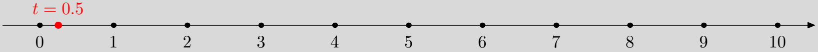

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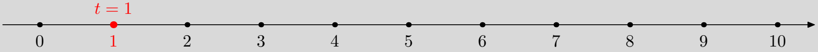

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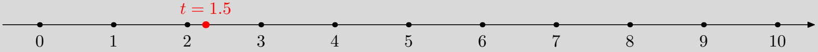

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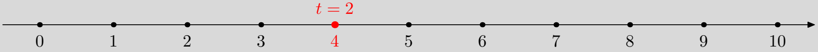

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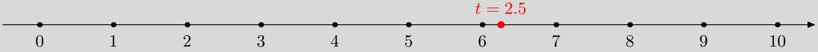

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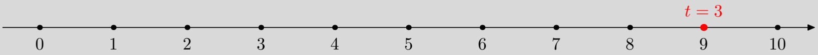

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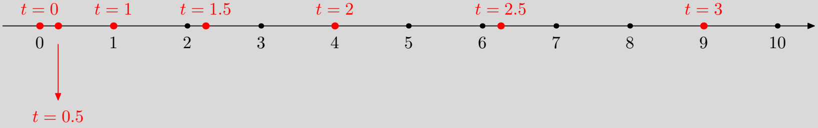

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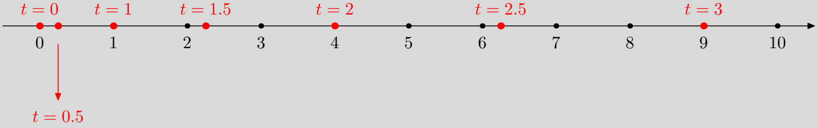


One can clearly see that the particle is **accelerating**.

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# Computing Average Speeds

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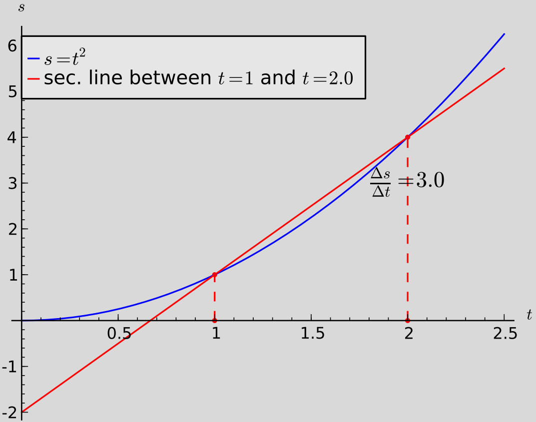

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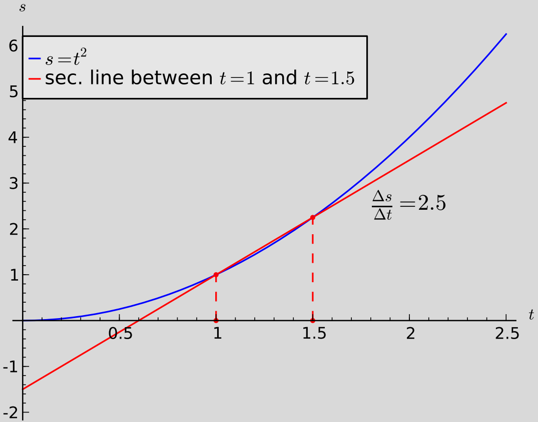

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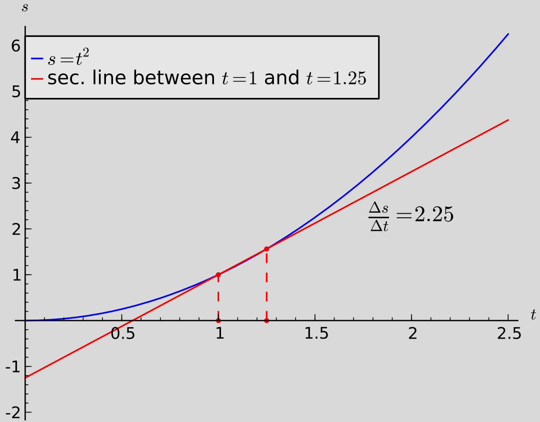

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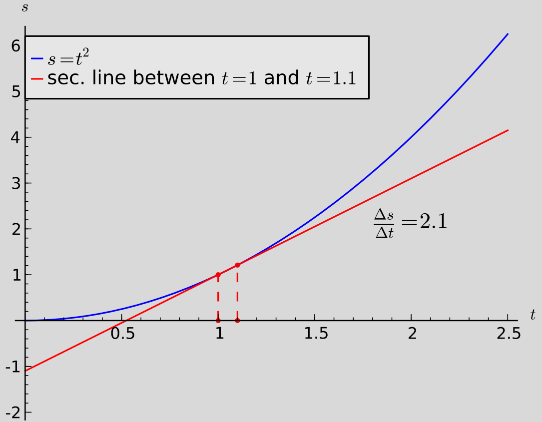

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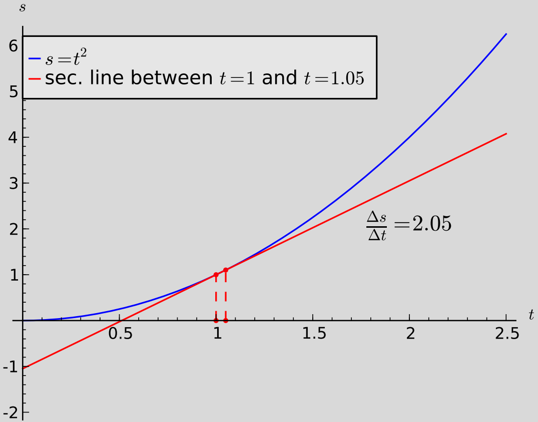

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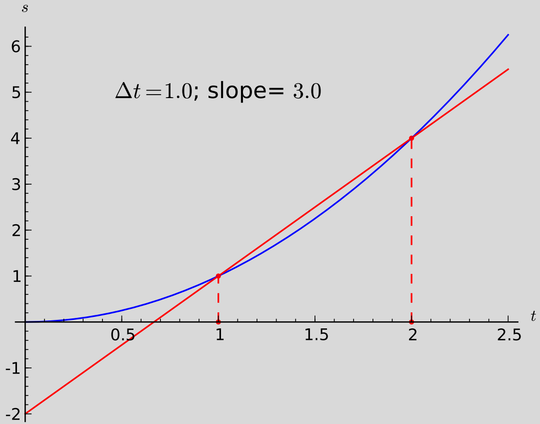

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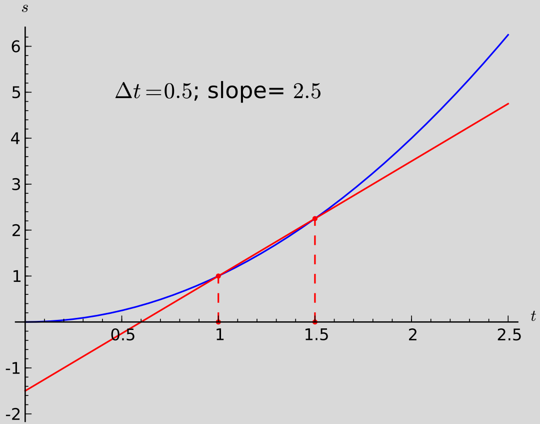

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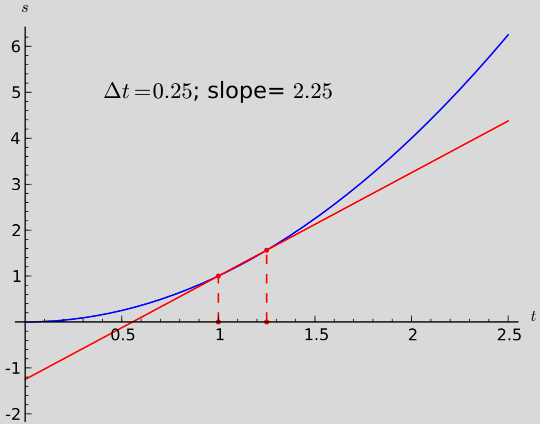

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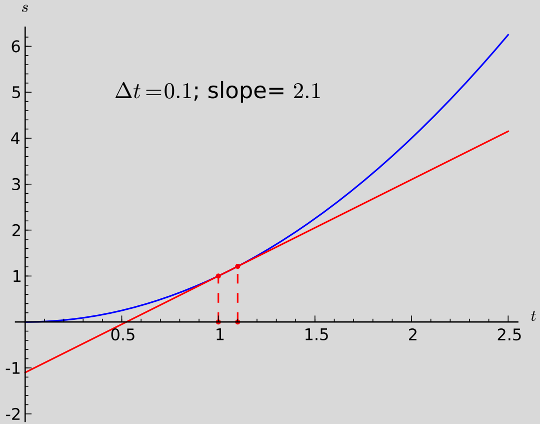

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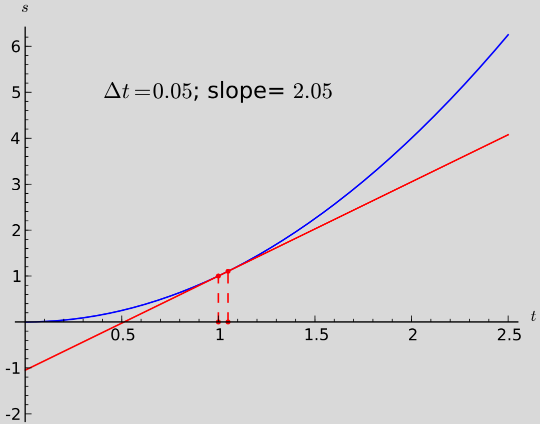

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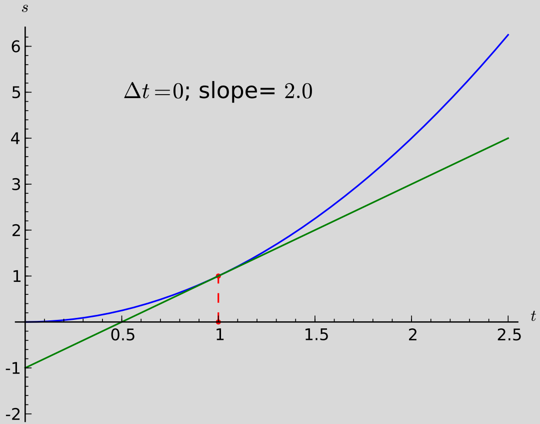

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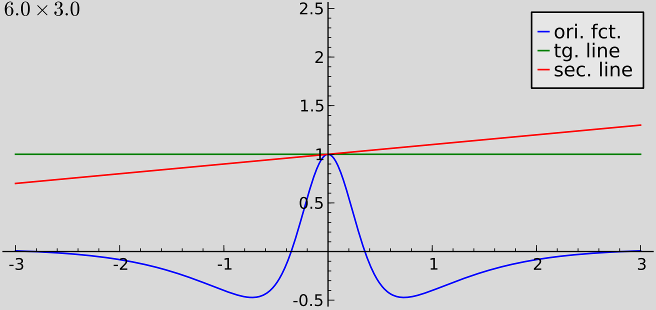

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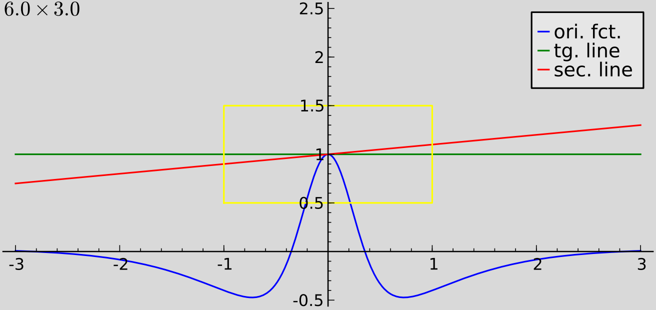

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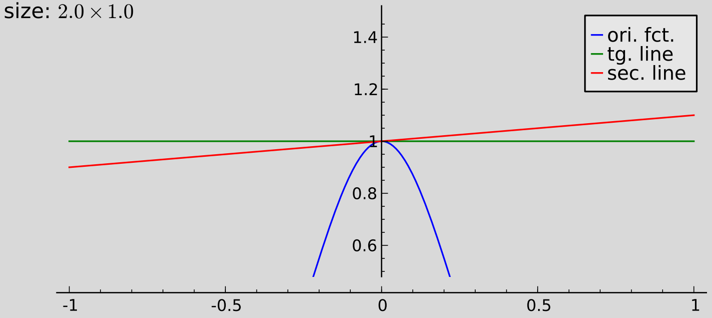

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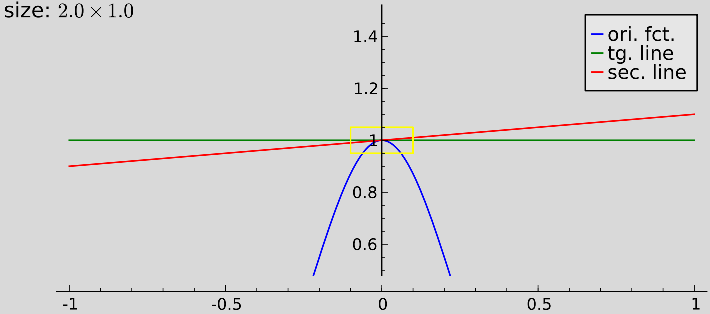

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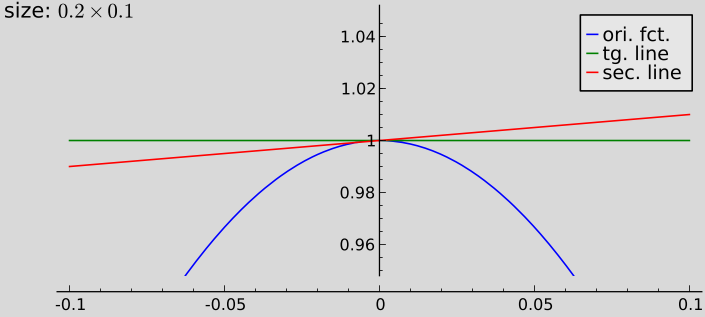

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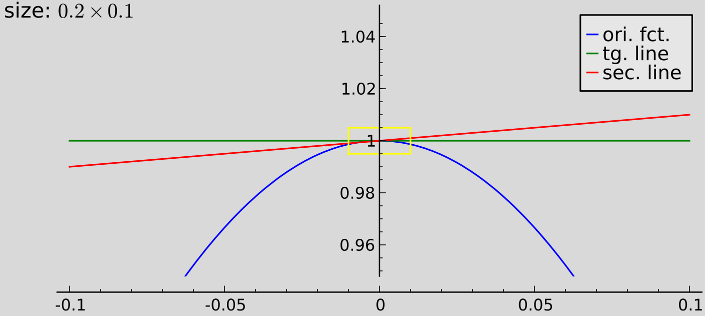

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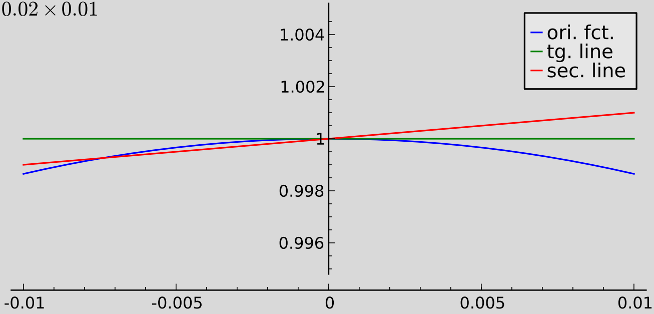

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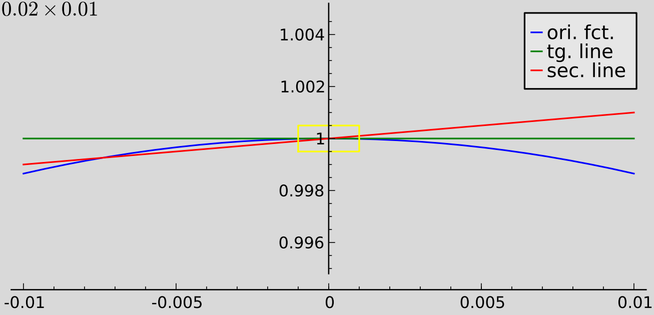

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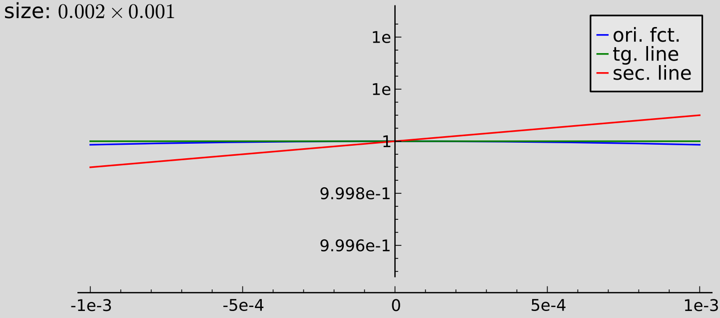

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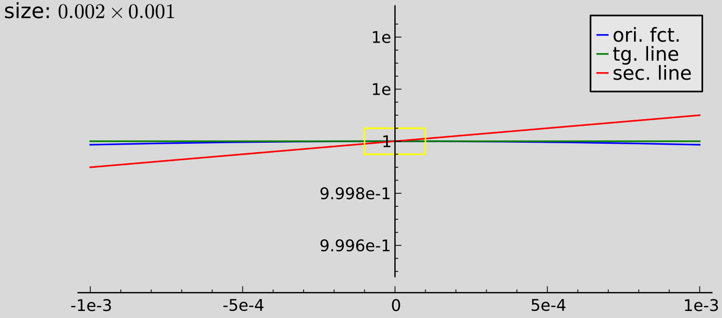

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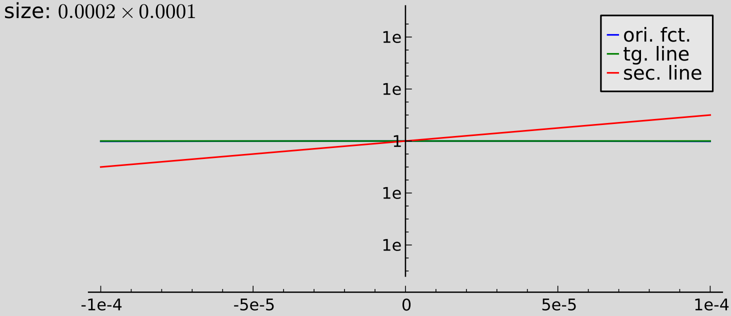

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In general, the rate of change of a function $f(x)$ at $x = x_0$ is the *slope* of the tangent line to the graph $y = f(x)$ at $x = x_0$. This tells us how fast is the $y$ value changing at $x = x_0$.

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Now, we can make $\Delta x = 0$ in the above expression, obtaining the answer: 1.

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We need the notion of **limit**!