October 7, 2013

Math 457

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Fall 2013

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 1

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 9 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work and justify your answers! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	15	
3	20	
4	20	
5	25	
Total	100	

1) [20 points] Consider the following permutations in S_7:

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 3 & 1 & 7 & 4 & 5 & 6 \end{pmatrix} \qquad \text{and} \qquad \tau = (1,7,4)(1,3,5)(2,6) \quad \text{[note it's not disjoint!].}$$

[**No need to show work for the items below!**]

(a) Write $\sigma \cdot \tau$ in the matrix representation [as σ was given].

(b) Write σ as a product of disjoint cycles.

(c) What is $|\sigma|$?

(d) Write σ as a product of transpositions.

(e) Find ρ such that $\rho\tau\rho^{-1} = (2,7,5)(2,3,1)(4,6)$. If there is no such ρ, say so and justify.

2) [15 points] Consider $D_6 = \{1, \rho, \rho^2, \ldots, \rho^5, \phi, \rho\phi, \rho^2\phi, \ldots, \rho^5\phi\}$ and its subgroup $H \stackrel{\text{def}}{=} \langle \rho^3, \phi \rangle$.

(a) Compute $(\rho^3\phi)^{231} \cdot (\rho^4\phi)^{-1} \cdot \rho^{601}$. [Your answer should be one of the listed elements above: ρ^i or $\rho^i\phi$, with $i \in \{0, \ldots, 5\}$.]

(b) List all the elements of H. [No need to justify or show work.]

(c) Is $H \lhd D_6$? [Justify!]

3) [20 points] Show that $A_5 \not\cong D_{30}$. [Here, it suffices to give a *structural* property that one of the groups has, but the other does not.]

4) [20 points] Let $N \lhd G$ and $\phi \in \mathrm{Aut}(G)$. Show that $\phi(N) \lhd G$.

5) In this problem, we will prove that if $p \neq 2$ is a prime and G is a group with $|G| = 2p$, then G has a normal subgroup of order p. [It is also true for $p = 2$ and it can be done directly. But here we will assume that $p \neq 2$.] **You can use a previous item even if you haven't proved it!**

(a) [10 points] Assume that there is no subgroup of order p. Prove that G is then Abelian. [**Hint:** Use an old HW problem.]

(b) [10 points] Still assuming that there is no subgroup of order p, show that G has a subgroup, say N, of order 2. Since G is Abelian (by (a)), we have that $N \lhd G$. Derive a contradiction by looking at G/N.

(c) [5 points] So, from the previous items, there is a subgroup of G, say H, of order p. Prove that $H \lhd G$.

Scratch: