December 12th, 2013

Math 141

Luís Finotti
Fall 2013

Name: ....

Student ID (last 6 digits): XXX- ....

TA recitation (check one):

Brian Allen: ☐ Anthony Zamberlan: ☐
Lindsey Fox (2:10): ☐ Lindsey Fox (3:40): ☐

Final

You have two hours to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 8 questions and 14 printed pages (including this one and two pages for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	15	
2	15	
3	10	
4	5	
5	25	
6	10	
7	10	
8	10	
Total	100	

1) Compute the following limits:

(a) [5 points] $\lim_{x \to 0} \frac{\sin(x^2)}{\ln(x+1)}$

(b) [5 points] $\lim_{x \to 0} x \ln(x^2)$

(c) [5 points] $\lim_{x \to 1} \frac{x^2+1}{(x-1)^2}$

2) Compute the derivatives of the following functions. [**No need to simplify your answers!**]

(a) [5 points] $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\sqrt{x}}{\cos(x)}\right)$

(b) [5 points] $\frac{\mathrm{d}}{\mathrm{d}x}\left(\ln(x^2+1)\cdot\arctan(x)\right)$

(c) [5 points] $\frac{\mathrm{d}}{\mathrm{d}x}\left((\sin(2^x))^5\right)$

3) Consider the curve given by the equation:

$$x^2 - y^2 = 1$$

(a) [5 points] Find the equation of the tangent line at the point $(2, \sqrt{3})$.

(b) [5 points] Is the curve concave up or concave down at $(2, \sqrt{3})$? [**Justify or you get zero!**]

4) [5 points] Find the equation of the line tangent to the curve given by

$$x(t) = t^2 - t$$

$$y(t) = \sqrt{t} + \frac{1}{t}$$

at $t = 1$.

5) Let $f(x) = x^3 - 4x$.

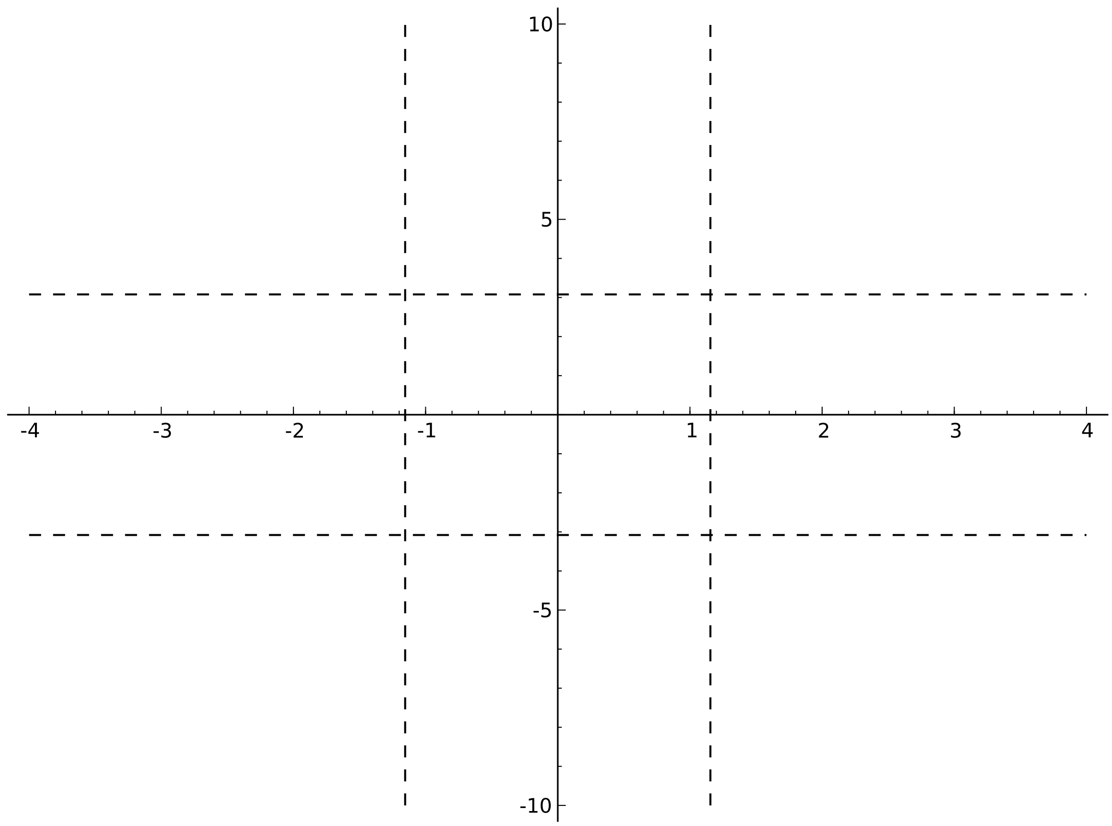
(a) [2 points] Find the x and y-intercepts of $f(x)$.

(b) [5 points] Give the intervals where $f(x)$ is increasing and decreasing and its critical points [if any], classifying them as local maximum, local minimum or neither.

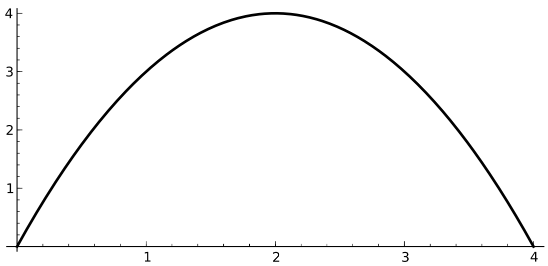
(c) [5 points] Give the intervals where $f(x)$ is concave up and concave down and its inflection points [if any].

(d) [5 points] Compute $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$

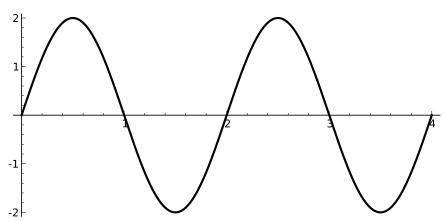
(e) [8 points] Sketch the graph of $f(x)$ below. [**Hint:** $2/\sqrt{3} = 2\sqrt{3}/3 \approx 1.1547$ and $f(2/\sqrt{3}) \approx -3.079$.]

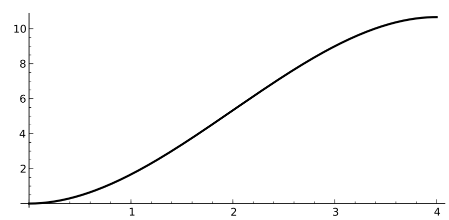


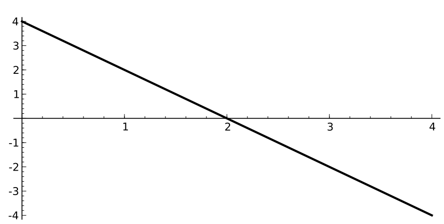
6) [10 points] Let $v(t)$ be the velocity of a particle moving along a straight line at time t. The picture below is the graph of $v(t)$:

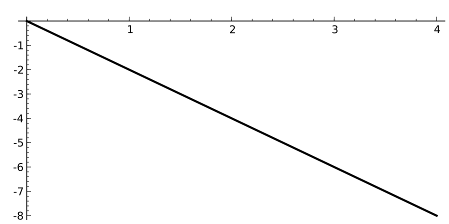


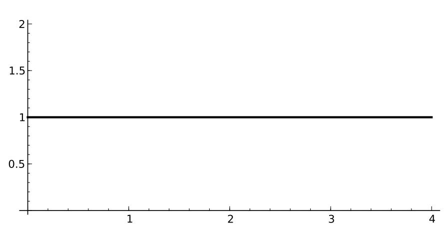
Which of the graphs below could represent the position of the particle and which could represent the acceleration? [Label the corresponding graphs clearly!]

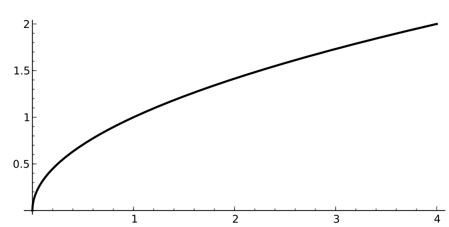




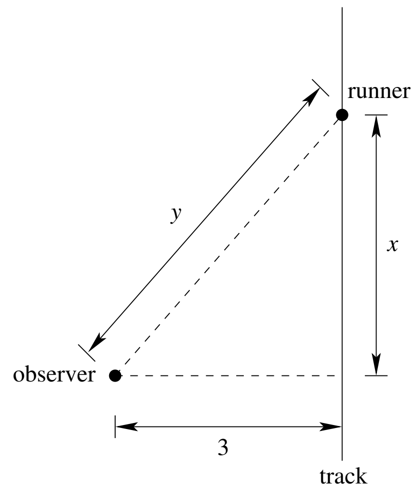








7) [10 points] A person in the audience watches a runner pass by in a long straight stretch. [See the picture below.] The person is 3 meters away from the track and the runner is running at a constant speed of 2.5 meters per second. [The runner is going *up* in the picture below, i.e., x is increasing.] How fast is the distance between the runner and the person in the audience [labeled y in the picture] increasing when the runner is 5 meters away [i.e., when $y = 5$]?



8) [10 points] You need to build a box with a *square base*, with both bottom and top [lid]. The box must have volume 16 cubic inches. The cost of the material for the sides of the box is 1 dollar per square inch, and the cost for the sturdier material for the top and bottom of the box is 2 dollars per square inch. What is the minimal cost to build this box?

Scratch: