September 28th, 2010

Math 251

Luís Finotti
Fall 2010

Name: ....

Student ID (last 6 digits): XXX- ....

Midterm 1

You have 60 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the last six digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 9 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! (Unless I say otherwise.) Correct answers without work will receive **zero**. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	40	
2	20	
3	20	
4	20	
Total	100	

1) *Quickies!* You don't need to justify your answers.

(a) If the reduced row echelon form of a square matrix A is *not* the identity matrix, what can you say about the number of solutions of $A\mathbf{x} = \mathbf{0}$?

(b) Let $A = \begin{bmatrix} 2 & 0 & 1 \\ -1 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ and $E = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Compute $A \cdot E$.

(c) Given that $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 3$, compute $\begin{vmatrix} 2d & 2e & 2f \\ a & b & c \\ g-3a & h-3b & i-3c \end{vmatrix}$.

(d) Give the equation of a plane perpendicular to $x + y - 2z = 3$.

(e) If $D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$, find A such that $D \cdot A = B$.

(f) Let $\mathbf{v} = (1, 2)$ and $\mathbf{a} = (1, 1)$. Find vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ such that $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$, with $\mathbf{v}_1$ having the same direction as $\mathbf{a}$ and $\mathbf{v}_2$ being perpendicular to $\mathbf{a}$.

(g) Let $A = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & 0 \\ 3 & 1 & 1 \end{bmatrix}$. Compute $(A \cdot B)^{\mathrm{T}}$.

(h) For what values of k is $A = \begin{bmatrix} 1 & -2 & 3 & 0 & 1 \\ 0 & k & 2 & -k & 3 \\ 0 & 0 & (k-1) & k^2 & 0 \\ 0 & 0 & 0 & (k+1)^2 & k \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ invertible?

2) Let

$$A = \begin{bmatrix} 1 & -1 & 3 & 2 \\ -2 & 1 & 5 & 1 \\ -3 & 2 & 2 & -1 \\ 4 & -3 & 1 & 4 \end{bmatrix}, \quad \mathbf{b}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad \text{and} \quad \mathbf{b}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \\ 1 \end{bmatrix}.$$

Find all solutions [if any] of the systems $A\mathbf{x} = \mathbf{b}_1$ and $A\mathbf{x} = \mathbf{b}_2$.

3) Let $A = \begin{bmatrix} 1 & 3 & 0 & 5 & 1 \\ 1 & 2 & 2 & -3 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 3 & -1 & 0 & 1 & 1 \\ 2 & 6 & 1 & 1 & 2 \end{bmatrix}$. Compute $\det(-A^3)$.

4) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$. Given that A is invertible with $A^{-1} = \begin{bmatrix} -40 & 16 & 9 \\ 13 & -5 & -3 \\ 5 & -2 & -1 \end{bmatrix}$, find B^{-1} where $B = \begin{bmatrix} 2 & 5 & 3 \\ 1 & 2 & 3 \\ 1 & 0 & 8 \end{bmatrix}$.

[**Note:** Observe that B is obtained from A by switching the first and second rows. You can use this and A^{-1} to compute B^{-1} in one second, *but you need to justify*! If you don't see it, or cannot justify, just compute B^{-1} directly.]

Scratch: