ERRATA

Abstract Algebra, Third Edition

by D. Dummit and R. Foote

(most recently revised on April 10, 2007)

These are errata for the Third Edition of the book. Errata from previous editions have been fixed in this edition so users of this edition do not need to refer to errata files for the Second Edition (on this web site). Individuals using the Second Edition, however, must make corrections from this list as well as those in the Second Edition errata files (except for corrections to text only needed in the Third Edition; for such text no reference to Second Edition page numbers is given below). Some of these corrections have already been incorporated into recent printings of the Third Edition.

page vi (2nd Edition p. vi)
from: 7.3 Ring Homomorphisms an Quotient Rings
to: 7.3 Ring Homomorphisms and Quotient Rings

page 31, The group $\mathbf{S_3}$ table
last line missing
add: $\sigma_6(1) = 3$, $\sigma_6(2) = 1$, $\sigma_6(3) = 2$ | (1 3 2)

page 33, Exercise 10, line 2 (2nd Edition p. 33, Exercise 10)
from: its least residue mod m when $k + i > m$
to: its least positive residue mod m

page 34, line 1 of Definition (2nd Edition p. 34, line 1 of Definition)
from: two binary operations
to: two commutative binary operations

page 39, Example 2, line -4
from: $ba = ab^{-1}$
to: $ba = a^{-1}b$

page 45, Exercise 22 (2nd Edition p. 46, Exercise 22)
from: is isomorphic to a subgroup (cf. Exercise 26 of Section 1) of S_4
to: is isomorphic to S_4

page 51, line -1 (2nd Edition p. 52, line -1)
from: see Exercise 1 in Section 1.7
to: see Exercise 4(b) in Section 1.7

page 71, Exercise 5 (2nd Edition p. 72, Exercise 5)
from: there are 16 such elements x
to: there are 8 such elements x

page 84, line 11 of Example 2 (2nd Edition p. 85, line 11 of Example 2)
from: By Proposition 2.6
to: By Theorem 2.7(1)

page 84, line -6 of Example 2 (2nd Edition p. 85, line -6 of Example 2)
from: By Proposition 2.5
to: By Theorem 2.7(3)

page 98, Figure 6
add: hatch marks to upper right and lower left lines of the central diamond (to indicate $AB/B \cong A/A \cap B$).

page 103, line 3 of Definition (2nd Edition p. 104, line 3 of Definition)
from: N_{i+1}/N_i a simple group
to: N_{i+1}/N_i is a simple group

page 114, line 3 in Proof of Proposition 2 (2nd Edition p. 116, line 3 of Proof)
from: $b \in G$
to: $g \in G$

page 132, Exercise 33, line -1 (2nd Edition p. 134, line -1 of Exercise 33)
from: See Exercises 6 and 7 in Section 1.3
to: See Exercises 16 and 17 in Section 1.3

page 132, Exercise 36(c) (2nd Edition p. 135, Exercise 36(c))
from: g and h lie in the center of G
to: g and h lie in the center of G and $g = h^{-1}$

page 139, Definition (1) (2nd Edition p. 141, Definition (1))
from: A group of order p^α for some $\alpha \geq 1$
to: A group of order p^α for some $\alpha \geq 0$

page 143, last line of first Example (2nd Edition p. 145, line -2)
from: Theorem 17
to: Proposition 17

page 148, Exercise 47(i) (2nd Edition p. 151, Exercise 47(i))
from: that has some prime divisor p such that n_p is not forced to be 1
to: for each prime divisor p of n the corresponding n_p is not forced to be 1

page 149, Exercise 54, line 4 (2nd Edition p. 151, line 4 of Exercise 54)
from: G/N acts as automorphisms of N
to: $G/C_G(N)$ acts as automorphisms of N

page 151, Exercise 6, line -2 (2nd Edition p. 153, line -2 of Exercise 6)
from: every pair of elements of D lie in a finite simple subgroup of D
to: every pair of elements of A lie in a finite simple subgroup of A

page 158, line 3 after the Definition (2nd Edition p. 160)
from: n-tuple
to: r-tuple

page 187, Exercise 23, line 4 (2^{nd} Edition p. 189, line 4 of Exercise 23)
from: from G into
to: from K into

page 191, Proposition 2 (2^{nd} Edition p. 193, Proposition 2)
from: nilpotence class at most $a-1$.
to: nilpotence class at most $a-1$ for $a \geq 2$ (and class equal to a when $a = 0$ or 1).

page 191, line 3 of the proof of Proposition 2 (2^{nd} Edition p. 193)
from: Thus if $Z_i(P) \neq G$
to: Thus if $Z_i(P) \neq P$

page 194, Theorem 8, line 4 (2^{nd} Edition p. 196, Theorem 8, line 4)
from: $Z_i(G) \leq G^{c-i-1} \leq Z_{i+1}(G) \quad \text{for all } i \in \{0, 1, \ldots, c-1\}$.
to: $G^{c-i} \leq Z_i(G) \quad \text{for all } i \in \{0, 1, \ldots, c\}$.

page 198, Exercise 18 (2^{nd} Edition p. 200, Exercise 18)
from: then $G'' = 1$
to: then $G'' = G'''$

page 209, Proposition 14(1)
from: $n_3 = 7$
to: $n_3 = 28$

page 216, line 4 after displayed steps (1) and (2) (2^{nd} Edition p. 217, line −3)
from: are equal if and only if $n = m$ and $\delta_i = \epsilon_i$, $1 \leq i \leq n$
to: are equal if and only if $n = m$, $r_i = s_i$ and $\delta_i = \epsilon_i$, $1 \leq i \leq n$

page 260, Exercise 40(iii) (2^{nd} Edition p. 261, Exercise 40(iii))
from: $R/\eta(R)$
to: $R/\mathfrak{N}(R)$

page 269, line 2 of Exercise 10(c) (2^{nd} Edition p. 270, Exercise 10(c))
from: then A may likewise
to: then P may likewise

page 269, line 2 of Exercise 11(d) (2^{nd} Edition p. 270, Exercise 11(d))
from: Prove that every ideal of
to: Prove that every nonzero ideal of

page 269, lines 1 and 2 of Exercise 11(e) (2^{nd} Edition p. 270, Exercise 11(e))
from: in the direct limit $\mathbb{Z}_p$ satisfying a_j^p
to: in the inverse limit $\mathbb{Z}_p$ satisfying a_j^{p-1}

page 282, second display (2^{nd} Edition p. 283, second display)
from: $0 < N(\frac{\alpha}{\beta}s - t) = \frac{(ay - 19bx - cq)^2 + 19(ax + by + cz)^2}{c^2} \leq \frac{1}{4} + \frac{19}{c^2}$

and so $(*)$ is satisfied with this s and t provided $c \geq 5$.

to: $0 < N(\frac{\alpha}{\beta}s - t) = \dfrac{(ay - 19bx - cq)^2 + 19(ax + by + cz)^2}{c^2} = \dfrac{r^2 + 19}{c^2} \le \dfrac{1}{4} + \dfrac{19}{c^2}$

and so $(*)$ is satisfied with this s and t provided $c \ge 5$ (note $r^2 + 19 \le 23$ when $c = 5$).

page 290, line 5 (2nd Edition p. 291, line 5)
from: is irreducible in $\mathbb{Z}[i]$
to: is irreducible in $\mathcal{O}$

page 312, Exercise 16(b) (2nd Edition p. 313, Exercise 16(b))
from: Prove that f
to: If $f(0) \neq 0$, prove that f

page 323, line −6
from: among the differences $S(g_i, g_j)$
to: among the remainders of the differences $S(g_i, g_j)$

page 332, Exercise 16, line 3
from: $(LT(g_1), \dots, LT(g_m), LT(S(g_i, g_j))$ is strictly larger than the ideal $(LT(g_1), \dots, LT(g_m))$. Conclude that the algorithm ...
to: $(LT(g_1), \dots, LT(g_m), LT(r))$ is strictly larger than the ideal $(LT(g_1), \dots, LT(g_m))$, where $S(g_i, g_j) \equiv r \bmod G$. Deduce that the algorithm ...

page 334, Exercise 43(a)
from: Use Exercise 30
to: Use Exercise 39

page 334, Exercise 43(b)
from: Use Exercise 33(a)
to: Use Exercise 42(a)

page 334, line 3 of Exercise 43(c)
from: ideal defined in Exercise 32,
to: ideal quotient (cf. Exercise 41),

page 350, line 2 of Exercise 4 (2nd Edition p. 331, Exercise 4)
from: $\varphi(\bar{k}) = ka$
to: $\varphi_a(\bar{k}) = ka$

page 372, Corollary 16(2), top line of commutative diagram
from: $M \times \cdots \times M_n \xrightarrow{\iota} M \otimes \cdots \otimes M_n$
to: $M_1 \times \cdots \times M_n \xrightarrow{\iota} M_1 \otimes \cdots \otimes M_n$

page 374, line 2 of second Remark (2nd Edition p. 355 line 2)
from: Section 11.6
to: Section 11.5

page 385, title of subsection following Proposition 26
from: Modules and $\mathrm{Hom}_R(D, \underline{\ \ })$
to: Projective Modules and $\mathrm{Hom}_R(D, \underline{\ \ })$

page 396, line –2 above Proposition 36 (2nd Edition p. 376)
from: Exercises 18 and 19
to: Exercises 19 and 20

page 398, Proof of Theorem 38 (2nd Edition p. 378)
from: Exercises 15 to 17
to: Exercises 15 and 16

page 399, line 8 (2nd Edition p. 379, line 22)
from: The map $1 \otimes \varphi$ is not in general injective
to: The map $1 \otimes \psi$ is not in general injective

page 401, line 2 of Example 1 (2nd Edition p. 381, line 2 of Example 1)
from: $\mathbb{Z}/2\mathbb{Z}$ not a flat module
to: $\mathbb{Z}/2\mathbb{Z}$ is not a flat module

page 403, Exercise 1(d) (2nd Edition p. 383, Exercise 1(d))
from: if β is injective, α and γ are surjective, then γ is injective
to: if β is injective, α and φ are surjective, then γ is injective

page 405, Exercise 15 (2nd Edition p. 385, Exercise 115)
change exercise to:

Let M be a left $\mathbb{Z}$-module and let R be a ring with 1.
(a) Show that $\mathrm{Hom}_{\mathbb{Z}}(R, M)$ is a left R-module under the action $(r\varphi)(r') = \varphi(r'r)$ (see Exercise 10).

(b) Suppose that $0 \to A \xrightarrow{\psi} B$ is an exact sequence of R-modules. Prove that if every $\mathbb{Z}$-module homomorphism f from A to M lifts to a $\mathbb{Z}$-module homomorphism F from B to M with $f = F \circ \psi$, then every R-module homomorphism f' from A to $\mathrm{Hom}_{\mathbb{Z}}(R, M)$ lifts to an R-module homomorphism F' from B to $\mathrm{Hom}_{\mathbb{Z}}(R, M)$ with $f' = F' \circ \psi$. [Given f', show that $f(a) = f'(a)(1_R)$ defines a $\mathbb{Z}$-module homomorphism of A to M. If F is the associated lift of f to B, show that $F'(b)(r) = F(rb)$ defines an R-module homomorphism from B to $\mathrm{Hom}_{\mathbb{Z}}(R, M)$ that lifts f'.]
(c) Prove that if Q is an injective $\mathbb{Z}$-module then $\mathrm{Hom}_{\mathbb{Z}}(R, Q)$ is an injective R-module.

page 423, line 3 of Exercise 9 (2nd Edition p. 403, Exercise 9)
from: If $\varphi|_W$ and $\widetilde{\varphi}$ are
to: If $\varphi|_W$ and $\bar{\varphi}$ are

page 426, line 2 of Exercise 21(b) (2nd Edition p. 406, Exercise 21(b))
from: $6z$
to: $+6z$

page 433, proof of Theorem 19, line 3
from: $= E_v(f) + \alpha E_g(v)$
to: $= E_v(f) + \alpha E_v(g)$

page 435, Exercise 1
change exercise to:

Let V be a vector space over F of dimension $n < \infty$. Prove that the map $\varphi \mapsto \varphi^*$ in Theorem 20 is a vector space isomorphism of $\mathrm{End}(V)$ with $\mathrm{End}(V^*)$, but is not a ring homomorphism when $n > 1$. Exhibit an F-algebra isomorphism from $\mathrm{End}(V)$ to $\mathrm{End}(V^*)$.

page 442, line −8 (2nd Edition p. 422, line −8)
from: $\varphi : M \to A$ is an R-algebra
to: $\varphi : M \to A$ is an R-module

page 479, last sentence of second paragraph (2nd Edition p. 459, second paragraph)
from: the degree of the minimal polynomial for A has degree at most n
to: the minimal polynomial for A has degree at most n

page 566, Example 7, first line after second display
from: we see that $\sigma_p^{p^n} = 1$
to: we see that $\sigma_p^n = 1$

page 584, Exercise 24 (2nd Edition p. 564, Exercise 24)
change exercise to:

Prove that the rational solutions $a, b \in \mathbb{Q}$ of Pythagoras' equation $a^2 + b^2 = 1$ are of the form $a = \dfrac{s^2 - t^2}{s^2 + t^2}$ and $b = \dfrac{2st}{s^2 + t^2}$ for some $s, t \in \mathbb{Q}$. Deduce that any right triangle with integer sides has sides of lengths $((m^2 - n^2)d,\ 2mnd,\ (m^2 + n^2)d)$ for some integers m, n, d. [Note that $a^2 + b^2 = 1$ is equivalent to $\mathrm{N}_{\mathbb{Q}(i)/\mathbb{Q}}(a + ib) = 1$, then use Hilbert's Theorem 90 above with $\beta = s + it$.]

page 585, Exercise 29(b) (2nd Edition p. 565, Exercise 29(b))
from: Prove that the element $t =$
to: Prove that the element $s =$

page 585, Exercise 29(c) (2nd Edition p. 565, Exercise 29(c))
from: Prove that $k(t)$
to: Prove that $k(s)$

page 654, Exercise 16 (2nd Edition p. 635, Exercise 16)
from: Prove that F does not contain all quadratic extensions of $\mathbb{Q}$.
to: Prove that F does contain all quadratic extensions of $\mathbb{Q}$. [One way is to consider the polynomials $x^3 + 3ax + 2a$ for $a \in \mathbb{Z}^+$.]

page 670, line 2 of Exercise 34 (2nd Edition p. 648, Exercise 34)
from: $\mathrm{Ass}_R(N) \subseteq \mathrm{Ass}_R(M)$
to: $\mathrm{Ass}_R(L) \subseteq \mathrm{Ass}_R(M)$

page 707, line 2 of Corollary 37(1) (2nd Edition p. 678, Corollary 29(1))
from: if and only if D contains no zero divisors of R
to: if and only if D contains no zero divisors or zero

page 721, line 4 after commutative diagram (2nd Edition p. 688, line 2)
from: By Proposition 38(1) [**2nd** Edition: By Proposition 30(1)]
to: By Proposition 46(1) [**2nd** Edition: By Proposition 36(1)]

page 728, Exercise 21, line 1
from: Suppose $\varphi : R \to S$ is a ring homomorphism
to: Suppose $\varphi : R \to S$ is a ring homomorphism with $\varphi(1_R) = 1_S$

page 754, line 2 of Exercise 8 (2nd Edition p. 720, Exercise 8)
from: Observe the
to: Observe that

page 756, line 1 of Proof of Proposition 5 (2nd Edition p. 722, Proof of Proposition 5)
from: $\nu(u) + \nu(v) = \nu(uv) = 1$
to: $\nu(u) + \nu(v) = \nu(uv) = \nu(1) = 0$

page 775, lines 1 to 3 of Exercise 24(d) (2nd Edition pp. 741–2, Exercise 24(d))
from: $P_3 = (3, 1 + \sqrt{-5}) = (3, 5 - \sqrt{-5})$ ...... [Check that $\sqrt{-10} = -(5 - \sqrt{-5})\omega/3$.]
to: $P_3 = (3, 1 - \sqrt{-5}) = (3, 5 + \sqrt{-5})$ ...... [Check that $\sqrt{-10} = (5 + \sqrt{-5})\omega/3$.]

page 781, bottom row of diagram (17.9) (2nd Edition p. 748, diagram (17.9))
from: $0 \longrightarrow \mathrm{Hom}_R(A, D) \longrightarrow$
to: $0 \longrightarrow \mathrm{Hom}_R(A', D) \longrightarrow$

page 793, line 4 of Exercise 11(c) (2nd Edition p. 760, Exercise 11(c))
from: projection maps $I \to I_i$
to: projection maps $I \to I/I_i$

page 794, Exercise 17 (2nd Edition p. 761, Exercise 17)
from: for any abelian group A
to: for any abelian group B

page 800, line −7 (2nd Edition p. 766, line −7)
from: $H^n(G, A) \cong Ext^n(\mathbb{Z}, A)$
to: $H^n(G, A) \cong Ext^n_{\mathbb{Z}G}(\mathbb{Z}, A)$

page 801, line 4 (2nd Edition p. 767, line 4)
from: 1 if n is odd
to: a if n is odd

page 812, Exercise 18(a) (2nd Edition p. 778, Exercise 18(a))
from: from $\mathbb{Z}/(m/d)\mathbb{Z}$ to $\mathbb{Z}/m\mathbb{Z}$ if n is odd, and from 0 to 0 if n is even, $n \geq 2$,
to: from 0 to 0 if n is odd, and from $\mathbb{Z}/(m/d)\mathbb{Z}$ to $\mathbb{Z}/m\mathbb{Z}$ if n is even, $n \geq 2$,

page 813, line 3 of Exercise 19 (2nd Edition p. 779, Exercise 19)
from: p-primary component of $H^1(G, A)$
to: p-primary component of $H^n(G, A)$

page 815, line 2 of Proposition 30 (2nd Edition p. 781)
from: group homomorphisms from G to H
to: group homomorphisms from G to A

page 816, line −13 (2nd Edition p. 782, line −13)
from: bijection between the elements of
to: bijection between the cyclic subgroups of order dividing n of

page 832, lines −10 and −14 (2nd Edition p. 798, lines −6 and −10)
from: L
to: K

page 853, line 4 of Exercise 17 (2nd Edition p. 819, Exercise 17)
from: Your proof . . .
to: Your proof that φ has degree 1 should also work for infinite abelian groups when φ has finite degree.

page 869, line −6 (2nd Edition p. 835, line −6)
from: the isotypic components of G
to: the isotypic components of M

page 885, Exercise 8 (2nd Edition p. 851, Exercise 8)
from: This table contains nonreal entries.
to: This table contains irrational entries.

page 893, line 4 (2nd Edition p. 859, line 4)
from: a proper, nontrivial subgroup of G
to: a proper, nontrivial normal subgroup of G

page 899, line 1 of item (3) (2nd Edition p. 865)
from: let Q_3 be a Sylow 11-subgroup of G
to: let Q_3 be a Sylow 13-subgroup of G

page 907, Exercise 1(a) (2nd Edition p. 873, Exercise 1(a))
from: a 3-tuple in $A \times A \times A$ maps to an ordered pair in $A \times A$
to: an ordered pair in $A \times A$ maps to a 3-tuple in $A \times A \times A$

page 912, line 6 (2nd Edition p. 878, line 6)
from: if $A \neq B$ or $C \neq D$
to: if $A \neq C$ or $B \neq D$