Math 455

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Name: . . . .

Student ID (middle 5 digits): XX XX

Final (Take-home part)

Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the middle five digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 5 questions and 6 printed pages (including this one).

You can use your notes and textbook, but you cannot talk or say *anything at all* about this exam to *anyone*. If you do, you will get an **F** in this course.

Since this is a take home, and you have plenty of time, I will particularly strict about how well written your answers are. **Points will be taken from messy solutions.** Also, I will be less generous with partial credit.

Show all work! Even correct answers without work may result in point deductions.

Good luck!

Question	Max. Points	Score
1	12	
2	12	
3	12	
4	12	
5	12	
Total	60	

1) Suppose that $|G| = 2p$, where p is a prime different from 2. Prove that either $G \cong C_{2p}$ or $G \cong D_{2p}$.

2) Let $H \triangleleft G$, $\bar{K} < G/H$, and

$$K \stackrel{\text{def}}{=} \{x \in G \ : \ x \in gH \text{ for some } gH \in \bar{K}\}$$

[i.e., K is the union of all cosets in $\bar{K}$].

(a) Prove that K is a subgroup of G containing H.

(b) Prove that $\bar{K} = \{kH \ : \ k \in K\}$.

3) Let $M, N \triangleleft G$.

(a) Prove that $(NM) < G$, $M \triangleleft (NM)$, and $(N \cap M) \triangleleft N$.

(b) Prove that $N/(N \cap M) \cong (NM)/M$.

4) Let G be an *Abelian group*, $H < G$ and $\phi : G \to H$ be a homomorphism such that $\phi(h) = h$ for all $h \in H$. Prove that $G \cong H \times \ker \phi$. [**Hint:** Remember that $\phi(g) = \phi(g')$ iff $g^{-1}g' \in \ker \phi$.]

5) Let R be a [not necessarily commutative] ring in which $a^2 = a$ for *all* $a \in R$.

(a) Prove that for all $a \in R$, we have $a = -a$.

(b) Prove that R is commutative. [**Hint:** Expand $(a + b)^2$ in the ring.]