November 16th, 2006

Math 455

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Fall 2006

Name: ....

Student ID (middle 5 digits): XX XX

Makeup Midterm 2

You are not allowed to talk about this exam AT ALL to ANYONE until the solution has been posted! If you fail to do so, you will get a zero and will be reported.

Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the middle five digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 7 printed pages (including this one and a page for scratch work in the end).

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**, especially for those who took the exam home.

Good luck!

Question	Max. Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) Let $G \stackrel{\text{def}}{=} C_4 \times C_8$. [As usual, C_n denotes the cyclic group of order n.] Let x and y denote the generators of C_4 and C_8 respectively, i.e., $C_4 = \langle x \rangle$ and $C_8 = \langle y \rangle$, and let $H \stackrel{\text{def}}{=} \langle (x, y^7) \rangle$.

(a) Give the elements of H explicitly.

(b) Describe G/H as a set. [In other words, give its elements.]

(c) To what group is G/H isomorphic? [Give a precise description, like S_3, Q_8, C_7, $C_2 \times C_2, \mathbb{Z}$, etc.]

2) Let $G = (0, \infty) \times \mathbb{R}$ act on $S \stackrel{\text{def}}{=} \mathbb{R}^2$ by: given $(r, t) \in G$ and $(x, y) \in S$,

$$f_{(r,t)}(x, y) \stackrel{\text{def}}{=} (rx, y + t).$$

(a) Prove that this indeed defines a group action.

(b) Describe the orbits of $(-\sqrt{2}, \pi)$ and $(0, 1)$.

[Continues on next page!]

(c) Describe the stabilizers of $(-\sqrt{2}, \pi)$ and $(0, 1)$.

3) Let G be a group with *normal* subgroups of orders 3 and 5. Prove that G has an *element* of order 15.
[If you don't think you can do this, you can try to do it with the assumption that G is Abelian. It's easier, but you will only get half of the credit.]

4) Let $G \stackrel{\text{def}}{=} \mathbb{Z} \times \mathbb{Z}$ and

$$H \stackrel{\text{def}}{=} \{(n, -n) \ : \ n \in \mathbb{Z}\}.$$

Prove that $H \triangleleft G$ and $G/H \cong \mathbb{Z}$.

Scratch: