November 6th, 2006

Math 455

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Fall 2006

Name: ....

Student ID (middle 5 digits): XX XX

Midterm 2

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the middle five digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	20	
2	25	
3	30	
4	25	
Total	100	

1) Let $G \stackrel{\text{def}}{=} \mathbb{Z}/36\mathbb{Z}$ and $H \stackrel{\text{def}}{=} \langle \bar{2} \rangle \cap \langle \bar{3} \rangle$. [As usual, $\bar{a}$ represents the coset $(a + 36\mathbb{Z})$ of $\mathbb{Z}/36\mathbb{Z}$.]

(a) Describe G/H as a set. [In other words, give its elements.]

(b) To what group is G/H isomorphic? [Give a precise description, like S_3, Q_8, C_7, $C_2 \times C_2, \mathbb{Z}$, etc.]

2) Let $G \stackrel{\text{def}}{=} \mathbb{R}^\times \times \mathbb{R}^\times$ act on $S \stackrel{\text{def}}{=} \mathbb{R}^2$ by: given $(a, b) \in \mathbb{R}^\times \times \mathbb{R}^\times$, and $(x, y) \in \mathbb{R}^2$,

$$f_{(a,b)}(x, y) \stackrel{\text{def}}{=} (ax, by).$$

(a) Prove that this indeed defines a group action.

(b) Describe the orbits of $(1, -3)$ and $(-\pi, 0)$ *geometrically*. [Like, "the circle of radius 3 and center at the origin", or "the vertical line passing though -2", or "the line $x = y$ minus the point $(1, 1)$", etc.]

[Continues on next page!]

(c) Describe the stabilizers of $(1, -3)$ and $(-\pi, 0)$.

3) Prove the following:

(a) Let G be a *finite* group. Prove that for all $a \in G$, we have $a^{|G|} = 1_G$.

(b) Let $H \lhd G$ with $[G : H] = n$. Prove that for all $a \in G$, we have $a^n \in H$. [**Note:** You **can** use item (a) in this proof, even if you didn't do it.]

4) Let G be an **Abelian** group and

$$\Delta \stackrel{\text{def}}{=} \{(g, g) \ : \ g \in G\}.$$

Prove that $\Delta \lhd G \times G$ and $(G \times G)/\Delta \cong G$.

Scratch: