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Math 455

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Fall 2006

Name: ....

Student ID (middle 5 digits): XX XX

Midterm 1

You have 50 minutes to complete the exam. Do all work on this exam, i.e., on the page of the respective assignment. Indicate clearly, when you continue your solution on the back of the page or another part of the exam.

Write your name and the middle five digits of your student ID number on the top of this page. Check that no pages of your exam are missing. This exam has 4 questions and 7 printed pages (including this one and a page for scratch work in the end).

No books, notes or calculators are allowed on this exam!

Show all work! Even correct answers without work may result in point deductions. Also, **points will be taken from messy solutions**.

Good luck!

Question	Max. Points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) Let $f_1, f_2 \in S_4$ be defined as:

$$f_1 : \begin{aligned} 1 &\mapsto 2 \\ 2 &\mapsto 1 \\ 3 &\mapsto 4 \\ 4 &\mapsto 3 \end{aligned} \qquad\qquad f_2 : \begin{aligned} 1 &\mapsto 3 \\ 2 &\mapsto 1 \\ 3 &\mapsto 2 \\ 4 &\mapsto 4 \end{aligned}$$

(a) Find $f_2 \circ f_1$ and f_2^{-1}. [Your answer should be given in the same form as f_1 and f_2 are given above.]

(b) Find the 4×4 matrix M_{f_2} associated to f_2. [You do not need to justify this one.]

(c) Find $\text{sign}(f_2)$.

2) Let G be a group and $a \in G$. Prove that the subset

$$C_a(G) \stackrel{\text{def}}{=} \{x \in G \, : \, ax = xa\}$$

is a *subgroup* of G.

[**Note:** $C_a(G)$ is *not* the center of G. The center has all elements of G that commute with *every* other element of G, while $C_a(G)$ has all elements of G that commute with a. But, the proof that $C_a(G)$ is a subgroup is *very* similar to the proof the the center is a subgroup, done in class.]

3) Let $\phi : G \to G'$ be an isomorphism and $H \lhd G$. Show that $\phi(H) \lhd G'$.

[**Note:** Remember that

$$\phi(H) \stackrel{\text{def}}{=} \{\phi(x) \ : \ x \in H\}.]$$

4) Let S be the set of all real numbers except -1, and define the operation "$*$" by:

$$a * b = a + b + ab.$$

Prove that $(S, *)$ is an *Abelian* group, in which the identity is 0 and the inverse of $a \in S$ is $\tilde{a} \stackrel{\text{def}}{=} -a/(1+a)$. [I am not using a^{-1} for the inverse a for you not to think that it is $1/a$. With the operation "$*$", the inverse of a is *not* a^{-1}, is the $\tilde{a}$ above.]

[**Note:** It might be helpful to prove it is commutative (i.e., Abelian) first. Then, start with the easy parts!]

You have one extra page for this solution.

Extra space for question 4.

Scratch: