

## Example of How To Use Function *lift*

The commands in MAGMA come after the prompt `>`. Answers come in the following line, without `>`.

Commands are followed by `;`.

Example:

```
> 2+2;
4
```

## Finite Fields in MAGMA

I'll call `F` the field of 7 elements:

```
> F:=GF(7);
(nothing printed)
```

Now `F` represents the field of 7 elements:

```
> F;
Finite field of size 7
```

To have "2" as an element of `F`, we type `F!2`:

```
> Parent(2);
Integer Ring
```

```
> Parent(F!2);
Finite field of size 7
```

Now we create a field `F125` of 125 elements. We'll call the generator of the multiplicative group `g`.

```
> F125<g>:=GF(5^3);
```

Let's see the elements of `F125`:

```
> Set(F125);
{ 1, g, g^2, g^3, g^4, g^5, g^6, g^7, g^8, g^9, g^10, g^11, g^12, g^13, g^14, g^15,
g^16, g^17, g^18, g^19, g^20, g^21, g^22, g^23, g^24, g^25, g^26, g^27, g^28, g^29,
g^30, 2, g^32, g^33, g^34, g^35, g^36, g^37, g^38, g^39, g^40, g^41, g^42, g^43, g^44,
g^45, g^46, g^47, g^48, g^49, g^50, g^51, g^52, g^53, g^54, g^55, g^56, g^57, g^58,
g^59, g^60, g^61, 4, g^63, g^64, g^65, g^66, g^67, g^68, g^69, g^70, g^71, g^72, g^73,
g^74, g^75, g^76, g^77, g^78, g^79, g^80, g^81, g^82, g^83, g^84, g^85, g^86, g^87,
g^88, g^89, g^90, g^91, g^92, 3, g^94, g^95, g^96, g^97, g^98, g^99, g^100, g^101,
g^102, g^103, g^104, g^105, g^106, g^107, g^108, g^109, g^110, g^111, g^112, g^113,
g^114, g^115, g^116, g^117, g^118, g^119, g^120, g^121, g^122, g^123, 0 }
```

We can find the minimal polynomial of `g` over the field of 5 elements. We'll call `x` the variable:

```

> f<x>:=MinimalPolynomial(g,GF(5));
> f;
x^3 + 3*x + 3

```

## Canonical Liftings

Now, assuming that we are running magma under the directory that contains the file "fctWitt.m", we load the file: > load "fctWitt.m"; Loading "fctWitt.m"  
 We lift now the curve  $x^3+a_0x+b_0$  finding  $a_1, b_1, x_1, P_1, a_2, b_2, x_2, P_2$  in that order (note that  $y_1=y_0P_1$  and  $y_2=y_0P_2$ ) by typing lift(a0,b0);

```

> lift(F!0,F!3);
0 5 6*x0^10 + 6*x0^4 + 3*x0
2*x0^12 + 4*x0^9 + 2*x0^6 + 4*x0^3 + 5
0 0 6*x0^91 + 3*x0^73 + x0^70 + 3*x0^67 + 6*x0^64 + 4*x0^61 + 3*x0^58 + 3*x0^55 +
    3*x0^52 + 6*x0^46 + 4*x0^43 + 3*x0^37 + 4*x0^34 + 2*x0^31 + 3*x0^28 + 6*x0^25 +
    5*x0^22 + 3*x0^19 + 6*x0^13 + 5*x0^10 + 3*x0^7 + 4*x0^4 + 2*x0
5*x0^114 + 3*x0^111 + 2*x0^108 + 2*x0^105 + x0^96 + 2*x0^93 + x0^87 + x0^84 + x0^81 +
    2*x0^75 + 3*x0^72 + x0^66 + 3*x0^60 + 4*x0^57 + x0^54 + 3*x0^51 + 3*x0^42 + 5*x0^39
    + 6*x0^30 + 4*x0^24 + 4*x0^21 + 2*x0^18 + 3*x0^15 + 2*x0^12 + x0^9 + 5*x0^6 +
    2*x0^3 + 2

```

We can give names:

```

> a1, b1, x1, P1, a2, b2, x2, P2 := lift(F!0,F!3);
> a1;
0
> b1;
5
> a2;
0

```

and so on...

If we only want  $x_1$  and  $x_2$

```

> _,_,x1,_,_,x2 := lift(F!0,F!3);
> x1;
6*x0^10 + 6*x0^4 + 3*x0
> x2;
6*x0^91 + 3*x0^73 + x0^70 + 3*x0^67 + 6*x0^64 + 4*x0^61 + 3*x0^58 + 3*x0^55 + 3*x0^52 +
    6*x0^46 + 4*x0^43 + 3*x0^37 + 4*x0^34 + 2*x0^31 + 3*x0^28 + 6*x0^25 + 5*x0^22 +
    3*x0^19 + 6*x0^13 + 5*x0^10 + 3*x0^7 + 4*x0^4 + 2*x0

```

Note that  $a_1, b_1, P_1, a_2, b_2, P_2$  were computed! They were just discarded in the end!

Another example

```

> a1, b1, x1, P1, a2, b2, x2, P2 := lift(F125!g,F125!3);

```

```

> a1;
g^87
> b2;
g^12
> x2;
g^21*x0^45 + g^25*x0^37 + g^43*x0^35 + g^118*x0^34 + g^89*x0^33 + g^82*x0^31 +
g^21*x0^30 + g^119*x0^29 + g^103*x0^27 + g^122*x0^26 + g^59*x0^24 + g^42*x0^23 +
g^88*x0^22 + g^84*x0^21 + g^18*x0^20 + g^105*x0^19 + g^54*x0^18 + g^47*x0^17 +
g^14*x0^16 + g^68*x0^15 + g^68*x0^14 + g^101*x0^13 + g^65*x0^12 + g^49*x0^11 +
g^44*x0^10 + g^44*x0^9 + g^29*x0^8 + g^88*x0^7 + g^101*x0^6 + g^17*x0^5 +
g^106*x0^4 + g^58*x0^3 + g^5*x0^2 + g^55*x0 + g^96

```

### Error Messages

If the Hasse invariant is 0 (supersingular curve):

```

> lift(F!25!0,F!25!1);
ERROR: hasse inv. = 0!

```

If the curve is singular:

```

> lift(F!1,F!2);
ERROR: singular curve

```

If the elements are not in the same field:

```

> lift(F!3,2);
ERROR: 3 and 2 are not in the same field

```

You should do:

```

> lift(F!3,F!2);
5 1 2*x0^10 + 5*x0^8 + 6*x0^6 + 4*x0^5 + 6*x0^4 + 3*x0^3 + 3*x0^2 + 6*x0 + 6
3*x0^12 + 3*x0^10 + 4*x0^9 + 6*x0^6 + 3*x0^5 + x0^4 + x0^2 + 5*x0 + 3
4 1 3*x0^91 + x0^77 + 5*x0^73 + 2*x0^71 + 5*x0^70 + x0^69 + 5*x0^68 + x0^67 + 6*x0^66 +
6*x0^64 + x0^63 + 5*x0^62 + 6*x0^61 + 4*x0^60 + 5*x0^58 + 3*x0^57 + 2*x0^56 + x0^55
+ x0^54 + 3*x0^52 + 4*x0^50 + 3*x0^48 + x0^47 + 4*x0^45 + 5*x0^43 + x0^41 + 3*x0^40
+ 4*x0^39 + x0^38 + x0^36 + 4*x0^35 + 6*x0^33 + 5*x0^29 + 2*x0^28 + x0^27 + 3*x0^26
+ 5*x0^25 + x0^23 + 6*x0^21 + 2*x0^20 + 5*x0^19 + 5*x0^18 + 6*x0^17 + 2*x0^16 +
4*x0^15 + x0^14 + 6*x0^13 + 5*x0^12 + 2*x0^11 + 3*x0^10 + 5*x0^9 + 5*x0^8 + 6*x0^7
+ 2*x0^6 + x0^5 + 6*x0^4 + 2*x0^3 + 4*x0^2 + 2*x0
6*x0^114 + 5*x0^112 + x0^111 + x0^110 + 6*x0^109 + 3*x0^108 + 2*x0^107 + 6*x0^106 +
6*x0^105 + 4*x0^96 + 4*x0^94 + 3*x0^93 + 5*x0^91 + x0^88 + x0^87 + 6*x0^86 + x0^85
+ 5*x0^83 + 6*x0^82 + 6*x0^81 + 6*x0^80 + x0^79 + 5*x0^77 + 2*x0^76 + 2*x0^75 +
5*x0^74 + 6*x0^73 + 3*x0^71 + 2*x0^70 + x0^69 + 6*x0^68 + 6*x0^67 + x0^66 + 5*x0^64
+ x0^63 + 3*x0^62 + 4*x0^61 + 3*x0^60 + 5*x0^59 + 3*x0^57 + x0^56 + 2*x0^55 +
2*x0^54 + x0^53 + x0^52 + x0^51 + 6*x0^50 + 5*x0^49 + 5*x0^48 + 3*x0^47 + 2*x0^44 +
x0^43 + 6*x0^42 + 3*x0^41 + 3*x0^40 + 2*x0^39 + x0^38 + 3*x0^37 + 5*x0^36 + 3*x0^35
+ 5*x0^34 + 6*x0^33 + 5*x0^32 + 6*x0^31 + 5*x0^30 + 2*x0^28 + 3*x0^27 + 5*x0^26 +

```

$$4*x^0^{25} + 3*x^0^{24} + 4*x^0^{23} + 6*x^0^{22} + 3*x^0^{19} + 2*x^0^{18} + 3*x^0^{17} + x^0^{16} + \\ 4*x^0^{15} + 6*x^0^{14} + 4*x^0^{13} + 5*x^0^{12} + 5*x^0^{11} + 2*x^0^9 + 3*x^0^8 + 6*x^0^5 + 4*x^0^4 \\ + 2*x^0^3 + 6*x^0^2 + 2*x^0 + 4$$

Again, not in the same field:

```
> lift(F125!2,F!3);
ERROR: 2 and 3 are not in the same field
```

Using field of characteristic 0:

```
> lift(2,3);
ERROR: Char. of the field is zero!
```

### More General Cases

Suppose we want to find the canonical lifting for every curve in characteristic 5.

We define:

```
> FF<a0,b0>:=FieldOfFractions(PolynomialRing(GF(5),2));
```

Note that a0 and b0 now are automatically associated to FF:

```
> Parent(a0);
Rational function field of rank 2 over GF(5)
Variables: a0, b0
```

Now we can do:

```
> lift(a0,b0);
(a0^3*b0^2 + b0^4)/a0
4*a0^6*b0 + a0^3*b0^3 + b0^5
4/a0*x0^7 + 4*b0/a0*x0^4 + a0*x0^3 + 3*b0*x0^2 + 3*b0^2/a0*x0 + a0*b0
1/a0*x0^8 + 2*x0^6 + 3*b0/a0*x0^5 + 2*a0*x0^4 + 3*b0*x0^3 + a0^2*x0^2 + 3*a0*b0*x0 +
3*a0^3 + 3*b0^2
(2*a0^36 + a0^33*b0^2 + a0^30*b0^4 + 3*a0^27*b0^6 + 2*a0^24*b0^8 + a0^18*b0^12 +
4*a0^12*b0^16 + 3*a0^9*b0^18 + 4*a0^6*b0^20 + 4*a0^3*b0^22 + 4*b0^24)/a0^11
a0^36*b0 + 4*a0^33*b0^3 + 3*a0^27*b0^7 + 4*a0^21*b0^11 + 4*a0^15*b0^15 + a0^12*b0^17 +
3*a0^6*b0^21 + b0^25
2/a0^10*x0^45 + 2/a0^6*x0^37 + (a0^6 + a0^3*b0^2 + b0^4)/a0^11*x0^35 + 2*b0/a0^6*x0^34
+ 3/a0^4*x0^33 + (4*a0^3 + 4*b0^2)/a0^6*x0^31 + 4*b0^5/a0^10*x0^30 +
4*b0^2/a0^5*x0^29 + (a0^3 + 3*b0^2)/a0^4*x0^27 + 2*b0/a0^2*x0^26 + (4*a0^3*b0 +
3*b0^3)/a0^4*x0^24 + (4*a0^6 + 3*a0^3*b0^2 + b0^4)/a0^5*x0^23 + (2*a0^3*b0^3 +
4*b0^5)/a0^6*x0^22 + (a0^3*b0^2 + b0^4)/a0^4*x0^21 + (2*a0^12*b0 + 4*a0^9*b0^3 +
2*a0^6*b0^5 + a0^3*b0^7 + b0^9)/a0^11*x0^20 + (a0^9 + 3*a0^6*b0^2 +
4*b0^6)/a0^6*x0^19 + (2*a0^6*b0 + a0^3*b0^3 + 4*b0^5)/a0^4*x0^18 + (2*a0^6 +
a0^3*b0^2 + b0^4)/a0^2*x0^17 + (3*a0^6*b0^3 + 4*a0^3*b0^5 + 3*b0^7)/a0^6*x0^16 +
(4*a0^9 + 4*a0^6*b0^2 + 3*b0^6)/a0^4*x0^15 + (4*a0^6*b0 + 2*b0^5)/a0^2*x0^14 +
(3*a0^9 + 4*a0^6*b0^2 + 3*a0^3*b0^4 + 2*b0^6)/a0^3*x0^13 + (3*a0^9*b0 + 2*a0^6*b0^3
+ a0^3*b0^5 + 2*b0^7)/a0^4*x0^12 + (2*a0^9*b0^2 + 3*a0^6*b0^4 + 4*a0^3*b0^6 +
```

$$\begin{aligned}
& 2*b^8/a^5*x^{11} + (2*a^6*b^3 + 4*a^3*b^5 + 3*b^7)/a^3*x^{10} + (3*a^9*b^2 \\
& + 2*a^6*b^4 + 4*a^3*b^6 + 4*b^8)/a^4*x^9 + (2*a^9*b^3 + 2*a^6*b^5 + \\
& 4*a^3*b^7 + b^9)/a^5*x^8 + (3*a^9*b^4 + 3*a^6*b^6 + a^3*b^8 + \\
& 2*b^{10})/a^6*x^7 + (3*a^9*b^3 + 3*a^3*b^7 + 2*b^9)/a^4*x^6 + (4*a^{21} + \\
& 3*a^{15}*b^4 + a^{12}*b^6 + 2*a^9*b^8 + a^6*b^{10} + 2*a^3*b^{12} + \\
& 2*b^{14})/a^{11}*x^5 + (a^9*b^5 + 3*a^6*b^7 + 4*a^3*b^9 + 2*b^{11})/a^6*x^4 + \\
& (4*a^9*b^4 + 4*a^6*b^6 + a^3*b^8 + 3*b^{10})/a^4*x^3 + (2*a^9*b^5 + \\
& 2*a^6*b^7 + 4*b^{11})/a^5*x^2 + (2*a^9*b^6 + 4*b^{12})/a^6*x + (4*a^{21}*b + \\
& a^{18}*b^3 + 4*a^{15}*b^5 + a^{12}*b^7 + a^9*b^9 + 3*a^6*b^{11} + 3*b^{15})/a^{10} \\
& 4/a^{10}*x^56 + 3/a^9*x^54 + 3*b/a^{10}*x^53 + 4/a^8*x^52 + 3*b/a^9*x^51 + \\
& 4*b^2/a^{10}*x^50 + 3/a^6*x^48 + (4*a^3*b^2 + 4*b^4)/a^{11}*x^46 + \\
& 4*b/a^6*x^45 + (3*a^6 + 3*a^3*b^2 + 3*b^4)/a^{10}*x^44 + (3*a^3*b^3 + \\
& 3*b^5)/a^{11}*x^43 + (2*a^6 + 4*a^3*b^2 + 4*b^4)/a^9*x^42 + (3*a^6*b + \\
& 3*a^3*b^3 + 4*b^5)/a^{10}*x^41 + (a^9 + 3*a^6*b^2 + 4*a^3*b^4 + \\
& 4*b^6)/a^{11}*x^40 + (2*a^6*b + 2*b^5)/a^9*x^39 + (2*a^9 + a^6*b^2 + \\
& 2*b^6)/a^{10}*x^38 + (2*a^6*b + 4*a^3*b^3 + b^5)/a^8*x^37 + (a^6*b^2 + \\
& 4*a^3*b^4 + 2*b^6)/a^9*x^36 + (4*a^6*b^3 + b^7)/a^{10}*x^35 + \\
& 3*b^4/a^5*x^34 + (4*a^3*b^3 + 3*b^5)/a^6*x^33 + (2*a^6 + \\
& 2*b^4)/a^4*x^32 + (a^{12}*b + a^9*b^3 + 3*a^6*b^5 + 2*a^3*b^7 + \\
& 2*b^9)/a^{11}*x^31 + (4*a^9 + 4*b^6)/a^6*x^30 + (2*a^{12}*b + 2*a^9*b^3 + \\
& 3*a^6*b^5 + 4*a^3*b^7 + 4*b^9)/a^{10}*x^29 + (2*a^{12}*b^2 + a^9*b^4 + \\
& 4*a^6*b^6 + 4*a^3*b^8 + 4*b^{10})/a^{11}*x^28 + (4*a^{12}*b + a^9*b^3 + \\
& 4*a^6*b^5 + 2*a^3*b^7 + 2*b^9)/a^9*x^27 + (4*a^{15} + 2*a^{12}*b^2 + \\
& 4*a^9*b^8 + 4*b^{10})/a^{10}*x^26 + (4*a^{15}*b + a^{12}*b^3 + 2*a^9*b^5 + \\
& 2*a^6*b^7 + 2*a^3*b^9 + 2*b^{11})/a^{11}*x^25 + (4*a^6 + 4*a^3*b^2 + \\
& 2*b^4)*x^{24} + (3*a^9*b + 3*a^6*b^3 + 2*b^7)/a^4*x^{23} + (3*a^9 + \\
& 3*a^6*b^2 + 4*a^3*b^4 + 2*b^6)/a^2*x^{22} + (2*a^9*b + b^7)/a^3*x^{21} + \\
& (3*a^9*b^2 + 4*a^3*b^6 + 2*b^8)/a^4*x^{20} + (4*a^7*b + a^4*b^3)*x^{19} + \\
& (3*a^6*b^2 + 3*a^3*b^4 + 2*b^6)*x^{18} + (3*a^9*b + 4*a^6*b^3 + 3*a^3*b^5 \\
& + 3*b^7)/a*x^{17} + (3*a^{12}*b^2 + 2*b^{10})/a^5*x^{16} + (a^{12}*b + 2*a^9*b^3 \\
& + 4*a^6*b^5 + 4*a^3*b^7 + 4*b^9)/a^3*x^{15} + (4*a^{15} + a^9*b^4 + \\
& 2*a^6*b^8 + 4*b^{10})/a^4*x^{14} + (2*a^{15}*b + 3*a^{12}*b^3 + 3*a^9*b^5 + \\
& a^6*b^7 + a^3*b^9 + 4*b^{11})/a^5*x^{13} + (4*a^{15} + 4*a^9*b^4 + 2*a^6*b^6 \\
& + 2*a^3*b^8 + 2*b^{10})/a^3*x^{12} + (2*a^{18}*b^3 + a^{15}*b^5 + a^{12}*b^7 + \\
& a^9*b^9 + 4*a^6*b^{11} + 4*b^{15})/a^{10}*x^{11} + (a^{18} + 4*a^{15}*b^2 + \\
& 3*a^9*b^6 + 2*a^6*b^8 + 4*a^3*b^{10} + 2*b^{12})/a^5*x^{10} + (a^{21}*b + \\
& 4*a^{18}*b^3 + 4*a^{12}*b^7 + 4*a^9*b^9 + 3*b^{15})/a^9*x^9 + (4*a^{24} + \\
& a^{21}*b^2 + 2*a^{18}*b^4 + a^{15}*b^6 + 4*a^{12}*b^8 + a^9*b^{10} + \\
& 3*b^{16})/a^{10}*x^8 + (a^{21}*b + a^{18}*b^3 + a^{15}*b^5 + 3*a^{12}*b^7 + \\
& 2*a^9*b^9 + 4*b^{15})/a^8*x^7 + (4*a^{24} + 2*a^{21}*b^2 + 3*a^{18}*b^4 + \\
& 3*a^{15}*b^6 + 4*a^{12}*b^8 + 3*a^9*b^{10} + 3*b^{16})/a^9*x^6 + (2*a^{24}*b + \\
& 2*a^{21}*b^3 + a^{18}*b^5 + 4*a^{15}*b^7 + 2*a^9*b^{11} + 4*b^{17})/a^{10}*x^5 + \\
& (2*a^{16} + a^{13}*b^2 + 4*a^4*b^8 + 4*a*b^{10})*x^4 + (a^{15}*b + 3*a^{12}*b^3 + \\
& 4*a^6*b^7 + 2*a^3*b^9 + b^{11})*x^3 + (4*a^{17} + 4*a^{14}*b^2 + 4*a^{11}*b^4 + \\
& a^8*b^6 + 4*a^5*b^8 + 2*a^2*b^{10})*x^2 + (a^{16}*b + 4*a^{13}*b^3 + \\
& 4*a^{10}*b^5 + 4*a^7*b^7 + 3*a^4*b^9 + a*b^{11})*x + 4*a^{18} + a^{12}*b^4 +
\end{aligned}$$

$$3*a0^9*b0^6 + a0^6*b0^8 + 3*b0^12$$

### Optional Parameters

**Timing:** If you want to check the times for each part of the calculation or to see what the computer is doing at the moment

```
> a1,b1,_,_,a2,b2 := lift(a0,b0 : tm:=true);
Comp. 1st coord.
Done with 1st coord.
Partial time = 0.019
Total time = 0.019
*****
Doing conversions and y0psm1, y0psp1
Done conv. and y0ps...
Partial time = 0.01
Total time = 0.029
*****
Comp. x2
Done comp. x2
Partial time = 0
Total time = 0.029
*****
comp witt prod and sum
Done comp. witt sum/prod
Partial time = 0
Total time = 0.029
*****
computing rhs2
Done comp. rhs
Partial time = 0.52
Total time = 0.549
*****
computing P2 and rem2
Done comp. P2 and rem2
Partial time = 0.271
Total time = 0.82
*****
computing M2 and v2
%%%%%%%%%%
M2 is 9 x 9
Number of var is 9
%%%%%%%%%%
done comp. M2 and v2
Partial time = 0.209
Total time = 1.029
```

```

*****
Solving the system
done solving the system
Partial time = 0.221
Total time = 1.25
*****
total time = 1.349
> a1;
(a0^3*b0^2 + b0^4)/a0
> a2;
(2*a0^36 + a0^33*b0^2 + a0^30*b0^4 + 3*a0^27*b0^6 + 2*a0^24*b0^8 + a0^18*b0^12 +
  4*a0^12*b0^16 + 3*a0^9*b0^18 + 4*a0^6*b0^20 + 4*a0^3*b0^22 + 4*b0^24)/a0^11

```

**Just Second Coordinate:** If you just care for a1, b1, x1, and P1 (in this case, a2, b2, x2, P2 are *not* computed at all, what makes it very fast):

```

> a1, b1, x1, P1 := lift(F!2,F!3 : ncoord:=1);
> a1;
2
> P1;
2*x0^12 + 6*x0^10 + 4*x0^9 + 2*x0^6 + 6*x0^5 + x0^4 + 5*x0^3 + 4*x0^2 + 2*x0 + 6

```

**Testing Results:** If you want to test the result (I'm pretty sure the program is right though... it worked for every example I tried.)

```

> lift(g^33,g^2 : test:=true);
%%%%%%%%%
test1 is OK
%%%%%%%%%
test2 is OK
%%%%%%%%%
1 g^36 g^29*x0^7 + 2*x0^4 + g^33*x0^3 + g^95*x0^2 + g^64*x0 + g^35
g^91*x0^8 + 2*x0^6 + 4*x0^5 + g^64*x0^4 + g^95*x0^3 + g^66*x0^2 + g^4*x0 + g^122
g^37 g^41 g^73*x0^45 + g^81*x0^37 + g^99*x0^35 + g^83*x0^34 + g^85*x0^33 + g^17*x0^31 +
g^114*x0^30 + g^25*x0^29 + g^36*x0^27 + g^91*x0^26 + g^104*x0^24 + g^35*x0^23 +
g^11*x0^22 + g^25*x0^21 + g^121*x0^20 + g^40*x0^19 + g^102*x0^18 + g^32*x0^17 +
2*x0^16 + g^46*x0^15 + g^22*x0^14 + g^37*x0^13 + g^23*x0^12 + g^86*x0^11 +
g^76*x0^10 + g^27*x0^9 + g^16*x0^8 + g^14*x0^7 + g^94*x0^6 + g^21*x0^5 + g^12*x0^4
+ g^39*x0^3 + g^116*x0^2 + g^27*x0 + g^3
g^104*x0^56 + g^44*x0^54 + g^13*x0^53 + g^46*x0^52 + g^46*x0^51 + g^108*x0^50 +
g^19*x0^48 + g^104*x0^46 + g^114*x0^45 + g^101*x0^44 + g^13*x0^43 + g^29*x0^42 +
g^64*x0^41 + g^113*x0^40 + g^76*x0^39 + g^48*x0^38 + g^61*x0^37 + g^35*x0^36 +
g^18*x0^35 + g^60*x0^34 + g^42*x0^33 + g^115*x0^32 + g^68*x0^31 + g^70*x0^30 +
g^52*x0^29 + g^19*x0^28 + g^78*x0^27 + g^36*x0^26 + g^32*x0^25 + g^113*x0^24 +
g^91*x0^23 + g^96*x0^22 + g^13*x0^21 + g^90*x0^20 + g^75*x0^19 + g^61*x0^18 +
g^12*x0^17 + g^56*x0^16 + g^41*x0^15 + g^6*x0^14 + g^56*x0^13 + g^80*x0^12 +
g^23*x0^11 + g^36*x0^10 + g^41*x0^9 + g^37*x0^8 + g^123*x0^7 + g^77*x0^6 +

```

$g^{111}x^5 + g^{107}x^4 + g^{16}x^3 + g^{69}x^2 + g^{13}x + g^{123}$   
 test 1 test for a1, b1, x1, P1, and test 2 test for a2, b2, x2, P2.

You can also mix all the options:

```
> a1,b1,x1,P1 := lift(a0,a0: tm:=true, test:=true, ncoord:=1);
Comp. 1st coord.
%%%%%%%%%%
test1 is OK
%%%%%%%%%%
Done with 1st coord.
Partial time = 0.01
Total time = 0.01
*****
```

or

```
> a1,b1,x1,P1,a2,b2,x2,P2 := lift(a0,a0: tm:=true, test:=true);
Comp. 1st coord.
%%%%%%%%%%
test1 is OK
%%%%%%%%%%
Done with 1st coord.
Partial time = 0.01
Total time = 0.01
*****
Doing conversions and y0psm1, y0psp1
Done conv. and y0ps...
Partial time = 0.011
Total time = 0.021
*****
Comp. x2
Done comp. x2
Partial time = 0.009
Total time = 0.031
*****
comp witt prod and sum
Done comp. witt sum/prod
Partial time = 0
Total time = 0.031
*****
computing rhs2
Done comp. rhs
Partial time = 0.559
Total time = 0.59
*****
computing P2 and rem2
```



```

Done comp. P2 and rem2
Partial time = 0.17
Total time = 0.76
*****
computing M2 and v2
%%%%%%%%%%
M2 is 9 x 9
Number of var is 9
%%%%%%%%%%
done comp. M2 and v2
Partial time = 0.121
Total time = 0.881
*****
Solving the system
done solving the system
Partial time = 0.06
Total time = 0.941
*****
%%%%%%%%%%
test2 is OK
%%%%%%%%%%
total time = 1.01
> a2;
2*a0^25 + a0^24 + a0^23 + 3*a0^22 + 2*a0^21 + a0^19 + 4*a0^17 + 3*a0^16 + 4*a0^15 +
4*a0^14 + 4*a0^13
> x2;
2/a0^10*x0^45 + 2/a0^6*x0^37 + (a0^2 + a0 + 1)/a0^7*x0^35 + 2/a0^5*x0^34 + 3/a0^4*x0^33
+ (4*a0 + 4)/a0^4*x0^31 + 4/a0^5*x0^30 + 4/a0^3*x0^29 + (a0 + 3)/a0^2*x0^27 +
2/a0*x0^26 + (4*a0 + 3)/a0*x0^24 + (4*a0^2 + 3*a0 + 1)/a0*x0^23 + (2*a0 +
4)/a0*x0^22 + (a0 + 1)*x0^21 + (2*a0^4 + 4*a0^3 + 2*a0^2 + a0 + 1)/a0^2*x0^20 +
(a0^3 + 3*a0^2 + 4)*x0^19 + (2*a0^3 + a0^2 + 4*a0)*x0^18 + (2*a0^4 + a0^3 +
a0^2)*x0^17 + (3*a0^3 + 4*a0^2 + 3*a0)*x0^16 + (4*a0^5 + 4*a0^4 + 3*a0^2)*x0^15 +
(4*a0^5 + 2*a0^3)*x0^14 + (3*a0^6 + 4*a0^5 + 3*a0^4 + 2*a0^3)*x0^13 + (3*a0^6 +
2*a0^5 + a0^4 + 2*a0^3)*x0^12 + (2*a0^6 + 3*a0^5 + 4*a0^4 + 2*a0^3)*x0^11 + (2*a0^6
+ 4*a0^5 + 3*a0^4)*x0^10 + (3*a0^7 + 2*a0^6 + 4*a0^5 + 4*a0^4)*x0^9 + (2*a0^7 +
2*a0^6 + 4*a0^5 + a0^4)*x0^8 + (3*a0^7 + 3*a0^6 + a0^5 + 2*a0^4)*x0^7 + (3*a0^8 +
3*a0^6 + 2*a0^5)*x0^6 + (4*a0^10 + 3*a0^8 + a0^7 + 2*a0^6 + a0^5 + 2*a0^4 +
2*a0^3)*x0^5 + (a0^8 + 3*a0^7 + 4*a0^6 + 2*a0^5)*x0^4 + (4*a0^9 + 4*a0^8 + a0^7 +
3*a0^6)*x0^3 + (2*a0^9 + 2*a0^8 + 4*a0^6)*x0^2 + (2*a0^9 + 4*a0^6)*x0 + 4*a0^12 +
a0^11 + 4*a0^10 + a0^9 + a0^8 + 3*a0^7 + 3*a0^5

```

## Minimal Liftings

If you want to compute the *minimal lift*:

```
> _,_,_,_,_,x2 :=lift(a0,a0 : test:=true, minimal:=true);
```

```

%%%%%%%%%%
test1 is OK
%%%%%%%%%%
%%%%%%%%%%
test2 is OK
%%%%%%%%%%
> x2;
2/a0^6*x0^37 + (a0 + 1)/a0^7*x0^35 + 2/a0^5*x0^34 + 3/a0^4*x0^33 + (4*a0 +
  4)/a0^4*x0^31 + 3/a0^5*x0^30 + 4/a0^3*x0^29 + (a0 + 3)/a0^2*x0^27 + 2/a0*x0^26 +
  (4*a0 + 3)/a0*x0^24 + (4*a0^2 + 3*a0 + 1)/a0*x0^23 + (2*a0 + 4)/a0*x0^22 + (a0 +
  1)*x0^21 + (2*a0^4 + 4*a0^3 + a0 + 1)/a0^2*x0^20 + (a0^3 + 3*a0^2 + 4)*x0^19 +
  (2*a0^3 + a0^2 + 4*a0)*x0^18 + (2*a0^4 + a0^3 + a0^2)*x0^17 + (3*a0^3 + 4*a0^2 +
  3*a0)*x0^16 + (2*a0^5 + 4*a0^4 + 3*a0^2 + 4)*x0^15 + (4*a0^5 + 2*a0^3)*x0^14 +
  (3*a0^6 + 4*a0^5 + 3*a0^4 + 2*a0^3)*x0^13 + (3*a0^6 + 2*a0^5 + a0^4 + 2*a0^3)*x0^12
  + (2*a0^6 + 3*a0^5 + 4*a0^4 + 2*a0^3)*x0^11 + (2*a0^6 + 3*a0^5 + 3*a0^4)*x0^10 +
  (3*a0^7 + 2*a0^6 + 4*a0^5 + 4*a0^4)*x0^9 + (2*a0^7 + 2*a0^6 + 4*a0^5 + a0^4)*x0^8 +
  (3*a0^7 + 3*a0^6 + a0^5 + 2*a0^4)*x0^7 + (3*a0^8 + 3*a0^6 + 2*a0^5)*x0^6 + (4*a0^10
  + 3*a0^8 + a0^7 + 2*a0^6 + 2*a0^4 + 2*a0^3)*x0^5 + (a0^8 + 3*a0^7 + 4*a0^6 +
  2*a0^5)*x0^4 + (4*a0^9 + 4*a0^8 + a0^7 + 3*a0^6)*x0^3 + (2*a0^9 + 2*a0^8 +
  4*a0^6)*x0^2 + (2*a0^9 + 4*a0^6)*x0 + 4*a0^12 + a0^11 + 4*a0^10 + a0^9 + a0^8 +
  3*a0^7 + a0^5

```

Note that this x2 has degree 37 and the one above (Teichmuller) has degree 45!

Another example:

```

> a1,b1,x1,P1,a2,b2,x2,P2:=lift(F!3,F!2 : minimal:=true);
> x2;
5*x0^73 + 2*x0^71 + 2*x0^70 + x0^69 + 5*x0^68 + x0^67 + 6*x0^66 + 6*x0^64 + 5*x0^62 +
  6*x0^61 + 4*x0^60 + 5*x0^58 + 3*x0^57 + 3*x0^56 + x0^55 + x0^54 + 3*x0^52 + 4*x0^50
  + 3*x0^48 + x0^47 + 4*x0^45 + 5*x0^43 + 3*x0^42 + x0^41 + 3*x0^40 + 4*x0^39 + x0^38
  + x0^36 + x0^35 + 6*x0^33 + 5*x0^29 + x0^27 + 3*x0^26 + 5*x0^25 + x0^23 + 2*x0^21 +
  2*x0^20 + 5*x0^19 + 5*x0^18 + 6*x0^17 + 2*x0^16 + 4*x0^15 + 6*x0^13 + 5*x0^12 +
  2*x0^11 + 3*x0^10 + 5*x0^9 + 5*x0^8 + 2*x0^6 + x0^5 + 6*x0^4 + 2*x0^3 + 4*x0^2 +
  2*x0

```

## Quiting

To quit Magma:

```
> quit;
```

Total time: 11.070 seconds

If you have any question or comment, feel free to email me.