Math 108.04 Syllabus – Fall 2007.

Course Description. First and second order ordinary differential equations with applications, Laplace transforms, series solutions and qualitative behavior, Fourier series, partial differential equations, boundary value problems, Sturm-Liouville theory. Intended primarily for for engineering and science students.
Text. Elementary Differential Equations and Boundary Value Problems by Boyce and DiPrima, 8th edition. Publisher: John Wiley & Sons, Inc. (ISBN 0-471-43338-1)
Prerequisites from Math 107: Sections 3.1–3.4 and 3.6 of Boyce and DiPrima.
Instructor. Fernando Schwartz, Room 225 Physics Building, fernando@math.duke.edu
http://www.math.duke.edu/~fernando.

Lesson 1 Section 1.1, Section 2.1. Introduction. Direction Fields, Linear equations; Method of Integration Factors.
§1.1: 11. §2.1: 1(abc),4(abc),14,20,28,33. Use Maple for #1(ab) and 4(ab).

Lesson 2 Section 2.2, begin Section 2.3. Modeling with First Order Equations; Differences Between Linear and Nonlinear Equations.
§2.2: 1,3,7,13(ac),16(ac),21,31(a,b),34(a,b),36(a,b).

Lesson 3 End Section 2.3, Section 2.4, Section 2.5 Theorems of Existence and Uniqueness of Solution; Autonomous equations.
§2.3: 2,8,9,10. §2.4: 7,9,14. §2.5: 3,22

Lesson 4 Section 2.6. Exact Equations and Integrating Factors.
§2.6: 1,5,7,11,12,18,21,25.

Lesson 5 Section 3.5, Section 3.7. Reduction of Order, Variation of Parameters.
§3.5: 23,28,33,38,39. §3.7: 3,5,8,15,18.

Lesson 6 **EXAM 1: Thursday 9/13.**

Lesson 7 Review Power Series. Section 5.1, begin Section 5.2. Series Solutions Near an Ordinary Point, Part 1.
§5.2: 1,5,8,12,13,14,18,19,21,25.

Lesson 8 Section 5.2, Section 5.3. Series Solutions Near an Ordinary Point, Part 2.
§5.2:2,10,15,23. §5.3: 3,8,11,15,22.

Lesson 9 Section 5.4, Begin Section 5.5. Regular Singular Points; Euler equations.
§5.4: 5,6,12,19,20. §5.5: 1,6.

Lesson 10 End Section 5.5, Sesction 5.6. Euler Equations; Series solution near a singular point.
§5.5: 18,19,23,24. §5.6: 3,7,8,11,14,16.

Lesson 11 Section 6.1, Section 6.2. Laplace Transform, Initial Value Problems.
§6.1: 2,3,5,6,9,26,27. §6.2: 1,2,3,8,9,13,14,16.

Lesson 12 Section 6.3, Section 6.4. Step Functions, Differential Equations with Discontinuous Forcing Functions.
§6.3: 1,4,6,8,10,11,15,16,19,20,27,29,31. §6.4: 3,5,9,12.

FALL BREAK: 10/6–9.

Lesson 13 Section 6.5, begin Section 6.6. Impulse Functions, The Convolution Integral.
§6.5: 1,4,9,12,13,17.

Lesson 14 End Section 6.6, Review.
§6.6: 1,6,9,11,13,14.

Lesson 15 **EXAM 2: Thursday 10/18.**

Lesson 16 Section 10.1. Two-Point Boundary Value Problems.
§10.1: 2,3,7,14,17,20.

Lesson 17 Review of 107. Begin Section 10.2. Review: Math 107 sections 9.1 – 9.3, including inner products, orthonormal bases, self-adjoint (Hermitian) matrices, etc.

Lesson 18 Section 10.2, Section 10.3. Fourier Series, The Fourier Convergence Thm.
§10.2: 4,6,8,9,16,18,29. §10.3: 2,4,13,14,15,17.

Lesson 19 Section 10.4. Even and Odd Functions.
§10.4: 3,5,6,7,12,16,17,35,36.

Lesson 20 Appendix A (p.649), begin Section 10.5. Derivation of the Heat Conduction Equation, Separation of Variables.
§10.5: 3,4,5,7,11,12,22.

Lesson 21 Section 10.6. Other Heat Conduction Problems.
§10.6: 2,8,11(a),12(a,b),15.

Lesson 22 Section 10.7. The Wave Equation: Vibrations of an Elastic string (including Derivation of the Wave Equation - Appendix B on p.653).
§10.7: 4,9,10.

Lesson 23 Section 10.8 and Review. Laplace's Equation.
§10.8: 2,7,8,10.

Lesson 24 **EXAM 3: Tuesday 10/20.**

THANKSGIVING: 11/21–25.

Lesson 25 Section 11.1. The Occurrence of Two-Point Boundary Value Problems.
§11.1: 2,3,4,5,8,10,19.

Lesson 26 Section 11.2. Sturm-Liouville Boundary Value Problems, Nonhomogeneous Boundary Value Problems.
§11.2: 1,4,7,8,11,13,14,15,27.

Lesson 27 Section 11.3. Nonhomogeneous Boundary Value Problems.
§11.3: 2,4,7,10,22

Lesson 28 Review for final exam.

FINAL BLOCK EXAM: Saturday 12/15, 9am–noon.

Warm-up Exercises

The following problems are not to be collected, but similar problems could be intermediate steps in the solutions of your homework problems, test problems or final exam problems.

________ (1) Complete the square of $2x^2 + x + 2$.

________ (2) Find all the values of the x in terms of union of intervals so that $|3x+1| \geq 4$.

________ (3) If $|f(x)| \leq 1$, $|g(x)| \leq 2$ for $x \in \mathbb{R}$, is $|3f(x) - 4g(x)| \leq 11$ on $\mathbb{R}$? Why?

________ (4) If $|f(x)| \leq 1$, $|g(x)| \leq 2$ and $|h(x)| \leq 3$ for $x \in \mathbb{R}$, is $|4f(x) + 5g(x) - 6h(x)| \leq 32$ on $\mathbb{R}$? Why?

________ (5) Solve for y from the equation $-\frac{1}{2}\ln|\frac{y}{x}+1| + \frac{1}{2}\ln|\frac{y}{x}-1| = \ln|x| + C$ where C is constant.

________ (6) Let $f(x) = 3x$ and $g(x) = \sin 2x$, compute $\int_0^t f(t-x)g(x)dx$ where $t \in \mathbb{R}$.

________ (7) Let $f(x) = |3x+1|$ and $g(x) = \sin 2x$, compute $\int_0^t f(t-x)g(x)dx$ where $t \in \mathbb{R}$.

________ (8) Compute $\int_0^\infty \frac{1}{(x^2+1)(x+1)}dx$.

________ (9) Find the antiderivatives $\int \frac{2x+3}{4-5x}dx$

________ (10) Compute $\int_0^2 (2x^3 - x + 1)\sin\frac{(2n-1)\pi x}{4}dx$ where $n = 0, 1, 2, \cdots$ and simplify your result as much as possible.

________ (11) Find A and θ so that $2\sin(3x) - 5\cos(3x) = A\cos(3x - \theta)$.

________ (12) Find the amplitude, angular frequency, phase angle and period of $y = 2\sin(3x) - 5\cos(3x)$.

________ (13) Differentiate $e^{x\sin x}$.

________ (14) Let $x = r\cos\theta$ and $y = r\sin\theta$. Rewrite $u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = 0$ in terms of u_{xx} and u_{yy}.

________ (15) Simplify $\sum_{n=0}^\infty e^{-nx}$ and determine the natural domain of the function represented by the given series.

________ (16) Find the radius of convergence of $\sum_{n=1}^\infty \frac{(-1)^n n^2}{3^n}(x+2)^n$.

________ (17) Find the first five nonzero terms of the power series represented by $(\sum_{n=0}^\infty \frac{(-1)^n}{(2n)!}x^{2n})(\sum_{n=1}^\infty (-2)^{n-1}x^n)$.