Exam #2 Math 32 – Fall 2005

Name:

I have adhered to the Duke Community standard in completing this test.

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signature

Q 1	/10
Q 2	/18
Q 3	/15
Q 4	/30
Q 5	/15
Q 6	/12
Total:	/100

Here's a list of formulas that may be of use to you. *Good luck!*

- $\frac{d}{dx}\sinh^{-1}(x) = \frac{1}{\sqrt{1+x^2}}$, $\frac{d}{dx}\cosh^{-1}(x) = \frac{1}{\sqrt{x^2-1}}$, $\frac{d}{dx}\tanh^{-1}(x) = \frac{1}{1-x^2}$
- If an integral involves $a^2 - u^2$, substitute $u = a\sin(\theta)$.
  If an integral involves $a^2 + u^2$, substitute $u = a\tan(\theta)$.
  If an integral involves $u^2 - a^2$, substitute $u = a\sec(\theta)$.

1. Solve the following initial value problems:

   (a) (5 pts.) $\frac{dy}{dx} = x^4 + y^2 x^4, \; y(0) = 1.$

   (b) (5 pts.) $\frac{dy}{dx} = \frac{y}{x \ln(x)}, \; y(e) = 2.$

2. Compute the following limits.

   (a) (6 pts.) $\lim_{x \to 0^+} x^x =$

(b) (6 pts.) $\lim_{x\to\infty} \frac{\tan^{-1}(x)}{e^x} =$

(c) (6 pts.) $\lim_{x\to\infty} \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)^x =$

3. Compute the following integrals:

   (a) (5 pts.) $\int \frac{\sinh^3(x)}{\cosh^7(x)} dx =$

(b) (5 pts.) $\int e^x \cos(x)dx =$

(c) (5 pts.) $\int \sinh^{-1}(x)dx =$

4. (a) (10 pts.) Write a factorization of $(x^3 + 1)$ as a product of a linear term and a quadratic term (Hint: find a root, then use long division).

(b) (10 pts.) Write a partial fraction decomposition of $\frac{1}{x^3+1}$

(c) (10 pts.) Compute the integral $\int \frac{1}{x^3+1} dx$

5. (a) (5 pts.) Prove that $\csc(x) = \frac{\sin(x)}{1-\cos^2(x)}$

(b) (5 pts.) Use part (a) and the fact that $\frac{1}{1-u^2} = \frac{1}{2}\left(\frac{1}{1+u}\right) + \frac{1}{2}\left(\frac{1}{1-u}\right)$ to prove that $\csc(x) = \frac{1}{2}\left(\frac{\sin(x)}{1+\cos x}\right) + \frac{1}{2}\left(\frac{\sin(x)}{1-\cos(x)}\right)$

(c) (5 pts.) Compute $\int \csc(x)dx$

6. Determine whether or not the following improper integrals converge. Evaluate those that do converge.

   (a) (6 pts.) $\int_{\pi}^{\infty} \frac{1}{(x-3)^{7/2}} dx$

   (b) (6 pts.) $\int_{0}^{\pi/2} \tan(x) dx$