## Growth vs. Reproduction:

## Background:

Why do organisms reproduce? This seems like a silly question at first until we consider whether or not organisms actually benefit from reproduction. Most organisms put their lives at risk to find a mate, and many invest considerable resources in giving birth and raising their offspring. But what benefit do they get out of this?

As humans, we think of having children as a benefit because they are our legacy: someone we can pass on our belongings and values to. But other than humans, it is unlikely that any organisms think about this. Presumably, from most organisms' perspective, anything that increases the risk of death is a bad thing (because once you're dead, you're dead).

So why do organisms reproduce? (In other words, why produce another life when it would seem better to just invest in your own life.) Consider mayflies. After being born, they grow as larvae at the bottom of bodies of water. After about a year, they metamorphose into winged adults. Strangely, the adults have no mouthparts, and so they only live for a few days on their stored food energy during which time they fly around looking for a mate. After finding a mate, the females lay their eggs and die, and the cycle begins again. If a mayfly does not find a mate within those few days, it will die and also won't leave any offspring.

Now, imagine a mayfly that "decides" not to metamorphose and reproduce, but instead "decides" to remain underwater as a larva and to keep on growing year-after-year. That would seem like a much better idea from that mayfly's point-of-view. (Indeed, there are species of cicadas that live and grow underground for 13 (and 17) years before finally emerging to mate.)

But imagine if all the mayflies in the population "decided" to do the same thing. What would happen to that population? Well, since everything eventually dies, this population will eventually go extinct. So this gives us the clue to what the benefit of reproduction is. It allows a population to survive even though all of its individuals will eventually die. In other words, populations that don't reproduce are no longer with us. (If your parents didn't have you, you wouldn't be here!)

So now that we understand why organisms reproduce, we may wonder: why do organisms do anything but reproduce? In other words, why aren't all organisms like mayflies? (After all, it would seem that the more a population reproduces, the more likely it is to survive and outlast other populations that don't reproduce as much.) The reason is that there are costs and risks associated with reproduction, and so ultimately a species will settle on an "optimum" balance of growth and reproduction, and this optimum will be determined by the species' particular life history and ecology.

## Experiment:

Understanding the tradeoffs between growth and reproduction is an important problem in the field of evolutionary ecology. To investigate this problem, you will use the tools of age-structured population models and Leslie matrices to find the optimum balance of growth and reproduction in a hypothetical insect population.

Like mayflies, this population consists of juveniles (up to one year old) and adults (one year and older). $90 \%$ of juveniles survive to adulthood.

First, you will consider a population where adults don't reproduce at all and $90 \%$ survive from year-to-year. The Leslie matrix for this population looks like this:

$$
L=\left[\begin{array}{cc}
0 & 0 \\
0.9 & 0.9
\end{array}\right]
$$

You will also consider 9 other populations where adults do reproduce but at the cost of decreased adult survival. The Leslie matrix for these populations looks like this:

$$
L=\left[\begin{array}{cc}
0 & k \sqrt{n}-n \\
0.9 & 0.9-0.1 n
\end{array}\right]
$$

where $n$ is a number between 1 and 9 and $k$ is a number calculated using your birthday as follows:

$$
k=6+\frac{\text { your birth month }+ \text { your birth date }}{20}
$$

For example, if your birthday is October $15, k=6+\frac{10+15}{20}=7.25$. Your value for $k$ should be between 6.1 and 8.15.

Along with the population that doesn't reproduce, you will analyze 9 other populations using the following values of $n: 1,2,3,4,5,6,7,8$, and 9. (The population that doesn't reproduce corresponds to $n=0$.)

1. Using Matlab, calculate the adult fertility, the adult survival rate, and the long-term growth rate (that is, the dominant eigenvalue) for each of these 10 populations. Create a table that presents this information.
Ex. (for $k=7.25$ )

| $n$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Adult <br> fertility | 0 | 6.25 | 8.253 |  |  |  |  |  |  |  |
| Adult <br> survival <br> rate | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 | 0.3 | 0.2 | 0.1 | 0.0 |
| Long- <br> term <br> growth <br> rate | 0.9 | 2.8052 | 3.0978 |  |  |  |  |  |  |  |

Also, use Matlab to plot long-term growth rate (as the dependent variable) vs. $n$ (as the independent variable). Finally, identify which population has the highest long-term growth rate. That population has achieved the optimum balance between growth and reproduction.

Next, we will add predation to each population (except for the one that doesn't reproduce) to see how that affects the optimum balance between growth and reproduction. To do that, we use the following matrix difference equation:

$$
\mathbf{x}_{n+1}=L \mathbf{x}_{n}-\mathbf{h}
$$

where $\mathbf{h}$ is the vector of predation.

The equilibrium for this equation is:

$$
\mathbf{x}_{e q}=(L-I)^{-1} \mathbf{h}
$$

where $I$ is the 2-by-2 Identity matrix.
Since (without predation) the populations are all growing, from the insects' perspective, the lower the value of $\mathbf{X}_{\text {eq }}$, the better. (This is because if the population starts out above the equilibrium, it will grow, but if it starts out below the equilibrium, it will decline to extinction.)
2. Using Matlab, calculate the total equilibrium population size (the sum of the elements of $\mathbf{X}_{\text {eq }}$ )
for each of the populations 1 through 9 using each of the following predation vectors:

$$
\mathbf{h}=\left[\begin{array}{c}
10000 \\
0
\end{array}\right],\left[\begin{array}{l}
7500 \\
2500
\end{array}\right],\left[\begin{array}{c}
5000 \\
5000
\end{array}\right],\left[\begin{array}{c}
2500 \\
7500
\end{array}\right],\left[\begin{array}{c}
0 \\
10000
\end{array}\right]
$$

For each predation vector, create a table that lists the total equilibrium population size for each population. Also, for each predation vector, use Matlab to plot total equilibrium population size (as the dependent variable) vs. $n$ (as the independent variable). Finally, for each predation vector, identify which population has the lowest total equilibrium population size. That population has achieved the optimum balance between growth and reproduction in that predation environment.
3. Use a word processing program (like MS Word) to type up the results of this experiment including: your value for $k$, all the tables and graphs you created, and a short analysis of the results including any hypotheses and conclusions about the optimum balance between growth and reproduction and the effect of different types of predation on the optimum balance.
HANDWRITTEN REPORTS WILL NOT BE ACCEPTED.
DO NOT INCLUDE YOUR MATLAB OUTPUT (EXCEPT FOR GRAPHS) WITH YOUR REPORT.

