

Some well-known difference equations and their equilibria

NAME	EQUATION	EQUILIBRIA	STABILITY
Arithmetic sequence	$\Delta x = d$ $x_{n+1} = x_n + d, (d \neq 0)$ $x_n = x_0 + d \cdot n$	none	
Geometric sequence	$\Delta x = k \cdot x_n$ $x_{n+1} = R \cdot x_n, (R = 1 + k)$ solution: $x_n = x_0 \cdot R^n$	$A = 0$ if $R \neq 1$	stable if $-1 < R < 1$ unstable if $R \leq -1$ or $R > 1$
		$A = \text{any } \#$ if $R = 1$	unstable
Exponential growth/decay	$\Delta N = k \cdot N_t$ $N_{t+1} = R \cdot N_t, (R = 1 + k)$ solution: $N_t = N_0 \cdot R^t$	$A = 0$ if $R \neq 1$	stable if $-1 < R < 1$ unstable if $R \leq -1$ or $R > 1$
		$A = \text{any } \#$ if $R = 1$	unstable
First order, linear difference equation with constant coefficients	$x_{n+1} = a \cdot x_n + b, (a \neq 1, b \neq 0)$ solution: $x_n = \left(x_0 - \frac{b}{1-a} \right) \cdot a^n + \frac{b}{1-a}$	$A = \frac{b}{1-a}$	stable if $-1 < a < 1$ unstable if $a \leq -1$ or $a > 1$
Logistic growth model	$\Delta N = \gamma \cdot \left(1 - \frac{N_t}{K} \right) \cdot N_t, (0 < \gamma < 3)$ $N_{t+1} = (1 + \gamma) \cdot N_t - \frac{\gamma}{K} \cdot (N_t)^2$ $N_{t+1} = R \cdot N_t - \frac{(R-1)}{K} \cdot (N_t)^2, (R = 1 + \gamma)$	$A = 0$	unstable
		$A = K$	stable if $0 < \gamma < 2$ oscillates if $1 < \gamma < 2$ unstable if $2 < \gamma < 3$
Gompertz growth model for tumors	$\Delta V = \left[\left(\frac{V_{\max}}{V_t} \right)^\gamma - 1 \right] \cdot V_t, (\gamma > 0)$ $V_{t+1} = (V_{\max})^\gamma \cdot (V_t)^{(1-\gamma)}$ solution: $V_t = V_{\max} \cdot \left(\frac{V_0}{V_{\max}} \right)^{[(1-\gamma)^t]}$	$A = V_{\max}$	stable if $0 < \gamma < 2$ oscillates if $1 < \gamma < 2$ unstable if $\gamma > 2$