Homework 1 for
UTK – M351 – Algebra I
Spring 2004, Jochen Denzler, MWF 10:10–11:00, Ayres 111

Problem 1:
I want you to collect, from your M300 notes, the *field axioms* for $\mathbb{R}$. However, I want you to do three things with them:

Firstly, whereever you encounter the set of real numbers $\mathbb{R}$, replace it by some yet unspecified set X. We will later give specific sets X and ask the question: Is each axiom still true when we replace $\mathbb{R}$ with X?

Secondly, I want you to organize the axioms in a table like the following:

There are functions $+$ and $\cdot$ defined on $X \times X$, with values in X such that the following axioms hold:

name	$+$	$\cdot$

Thirdly, I want you to help yourself in memorizing the names of these axioms; this is not a problem about math, but about language: find words in common language vocabulary that use the same roots as are used in the names of these axioms, try to abstract the unifying meaning of the root from these commonly known words, and employ these to understand *why* mathematicians have given these very names to the axioms. (Write these on scratch paper, for classroom discussion, not for grade: you may find it difficult, and I don't want you to contaminate this sheet with wrong answers.)

Problem 2:
Explore and discover...

In the above list, check each axiom for validity, if X is

(a) The set of integers, $\mathbb{Z}$

(a1) The set of odd integers

(a2) The set of even integers

(b) The set of rationals, $\mathbb{Q}$

(c) The set of 2×2 matrices with real entries (where $+$ and $\cdot$ denote addition and multiplication of matrices)

(d) The set P of polynomials in the variable x, with rational coefficients: in case you don't remember from calculus or 251 what a polynomial is, you must now refill your fragmentary memory by asking questions.

(e) The set $\{E, O\}$, where the following rules define $+$ and $\cdot$: $E + E = E$, $E + O = O + E = O$, $O + O = E$. $E \cdot E = E$, $E \cdot O = O \cdot E = E$, $O \cdot O = O$. I want you to discover also the *meaning* of these rules, i.e., how did I come up with them. Hint: the names E and O are a hint.

(f) *You* invent this problem: Invent a set with *three* elements to make this problem analogous to the previous one. Part of your job is to invent good names for these three elements.

(g) Same task as before; but this time we want a set of *four* elements. You should now focus on which axioms we are losing, as compared to (e) and (f). Any hunch what feature of the numbers 2, 3, 4 is decisive?